Math 215a Homework #5, Due Tuesday 11/15 at 9:40 AM

1. Hatcher section 2.2 exercise 8.

2. Hatcher section 2.2 exercise 17.

3. Hatcher section 2.2 exercises 20, 21, 22. (These should be quick.)

4. Let X and Y be compact surfaces, and let $f : X \to Y$ be a degree d branched cover[1]. Let r denote the sum of the orders of the ramification points in X. Prove the *Riemann-Hurwitz formula*

$$\chi(X) = d \cdot \chi(Y) - r.$$

5. Show that if $f : S^n \to S^n$ has degree d, then $f_* : H_n(S^n; G) \to H_n(S^n; G)$ is multiplication by d.

6. Find an example of spaces X, Y and maps $f, g : X \to Y$ which induce the same map $H_n(X) \to H_n(Y)$ for all n, but which induce different maps $H_n(X; G) \to H_n(Y; G)$ for some n and G. (*Hint:* take $X = \mathbb{RP}^2$ and $Y = S^2$.) Why doesn't this contradict the universal coefficient theorem?

7. How difficult was this assignment? (1 = very easy, 5 = very hard)

[1]This means that for every $p \in X$, there are identifications of neighborhoods of p and $f(p)$ with $\mathbb{C}$ identifying p and $f(p)$ with 0, such that under these identifications, $f(z) = z^n$ near the origin where n is a positive integer. So if $n = 1$ then f is a local homeomorphism at p; while if $n > 1$ then p is called a *ramification point* of *order* $n - 1$, and $f(p)$ is a *branch point*. Over the complement of the branch points, f is a covering, and d is its degree. Ordinarily one requires X and Y to be oriented and the above identifications to be orientation-preserving, but that it is not necessary for this problem.