Math 215a Homework #3, Due Thursday 10/13 at 9:40 AM

0. Prove that a short exact sequence of chain complexes induces a long exact sequence on homology. Don't hand this in.

1. Hatcher, section 2.1, exercise 8.

2. Hatcher, section 2.1, exercise 12. Also show that composition of chain maps induces a well-defined map on equivalence classes.

3. Prove carefully that if

$$0 \longrightarrow \mathbb{Z}^{n_1} \xrightarrow{f_1} \mathbb{Z}^{n_2} \xrightarrow{f_2} \cdots \xrightarrow{f_{k-1}} \mathbb{Z}^{n_k} \longrightarrow 0$$

is exact, then $\sum_{i=1}^{k}(-1)^i n_i = 0$.

For the next three problems, use the Mayer-Vietoris sequence.

4. Compute the homology of the space obtained by taking three copies of D^n and identifying their boundaries with each other.

5. Compute the homology of a genus g surface with n disjoint discs removed.

6. Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}_2\,\mathbb{Z}$; this induces a self-homeomorphism ϕ_A of $\mathbb{R}^2/\mathbb{Z}^2 = S^1 \times S^1$. Let Y be the 3-manifold[1] obtained by taking two copies of $S^1 \times D^2$ and identifying the boundary tori via ϕ_A. Compute the homology of Y, in terms of A.

7. How difficult was this assignment? (1 = very easy, 5 = very hard)

[1]For example, if A is the identity matrix, then $Y \simeq S^1 \times S^2$; if $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ then $Y \simeq S^3$. Can you see why?