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Continuity of the integral as a function of the domain

Abstract

We present here the fundamentals of a theory of domains that offers unifying techniques and terminology for a number of different fields. Using direct, geometric methods, we develop integration over p-dimensional domains in n-dimensional Euclidean space □^{n}, replacing the method of parametrization of a domain with the method of approximation in Banach spaces. We prove basic results needed for a theory of integration — continuity of the integral as a function of its domain and integrand (Corollary 4.8) and a generalization of Stokes's theorem (Corollary 4.14).

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Continuity of the Integral as a Function of the Domain

By *Jenny Harrison*

ABSTRACT. *We present here the fundamentals of a theory of domains that offers unifying techniques and terminology for a number of different fields. Using direct, geometric methods, we develop integration over p-dimensional domains in n-dimensional Euclidean space $\mathbb{R}^n$, replacing the method of parametrization of a domain with the method of approximation in Banach spaces. We prove basic results needed for a theory of integration — continuity of the integral as a function of its domain and integrand (Corollary 4.8) and a generalization of Stokes's theorem (Corollary 4.14).*

1. Introduction

In 1949, Whitney [15] initiated the study of how the integral of a fixed differential form ω defined in a neighborhood of a domain M in $\mathbb{R}^n$ varies as a function of the position of M. The concept of a domain of integration that emerges extends beyond the usual notion for polyhedra to limits of polyhedra in Banach spaces. Whitney defined the sharp and flat norms for his study. However, it turns out that the boundary operator is not continuous w.r.t. the sharp norm and the Banach space of flat forms is not closed under the Hodge star operator. (See Example 4.3.) Thus, Green's theorem cannot be stated for either sharp or flat domains and there is no Laplace operator Δ defined for either of them.

Our approach bifurcates from Whitney's starting with his first definitions. The one parameter family of norms of this paper are a blend of Whitney's flat norm [14] and Lebesgue's L^1 norm [12] (see Sections 2 and 4). Daniell [2] completed step functions w.r.t the L^1 norm to create the Banach space of L^1 functions. Lebesgue theory was satisfying in that all naturally arising integrable functions were represented in this Banach space. The author's search has been for Banach space representatives of all naturally arising domains.

The Banach spaces of domains introduced in this paper are denoted $\mathcal{A}_p^{r,\alpha}$, $r \in \mathbb{Z}$, $r \geq -1$, $0 < \alpha \leq 1$, and are taken with respect to norms $|P|_{r,\alpha}$, initially defined for polyhedra P. The *mass* of a polyhedral chain or p-dimensional Hausdorff measure coincides with $|P|_0$. When the dimension p is understood, we write $\mathcal{A}^{r,\alpha} = \mathcal{A}_p^{r,\alpha}$ and call its elements *p-chainlets*. We identify the integer r with $(r-1, 1)$ and order pairs (r, α) lexicographically. (Throughout the paper we assume $0 < \alpha \leq 1$. The reader is advised to set $\alpha = 1$ for the first reading.)

For $(r, \alpha) > 0$, the $C^{r,\alpha}$ differentiability class of a differential form is defined in the standard way, with norm $\|\omega\|_{C^{r,\alpha}}$. The exterior derivative of a smooth differential form is the standard analytic

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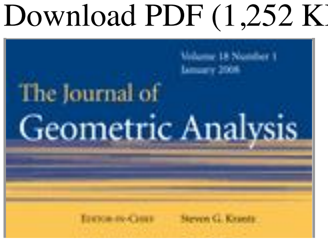

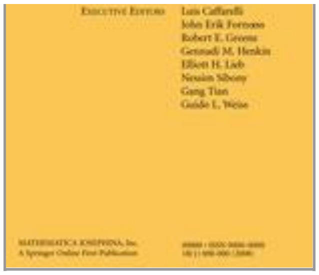
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- Jenny Harrison [1]

Author Affiliations

- 1. Department of Mathematics, University of California, Berkeley

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