

ON SOCIAL JUSTICE IN MATH AND THE MATH OF SOCIAL JUSTICE

ALEXANDER GIVENTAL

Do you remember the [publication hoax](#) by Peter Boghossian, James Lindsay, and Helen Pluckrose intended to expose low academic standards in what they called “grievance studies”? Apparently we are up to a new one, [arXiv:2308.13750](#) (by a large group of authors whom I don’t want to list here, because some of them are students possibly used blindly by their advisers), this time on “social justice”, which caricatures the sociology of the mathematics research community.

The paper was brought to my attention by the leader of a BLM reading group I was attending, who apparently was caught off guard by the ideologically correct title: *Quantifying Inequities and Documenting Elitism in PhD-granting Mathematical Sciences Departments in the United States*, and mistook the paper for genuine scholarship. But at any closer look the authors’ logical and factual missteps seem so childish, and the conclusions are so grotesque, that I can’t help but assume that a hoax was the goal (and apologize in advance if, however implausibly, I am mistaken about this).

How are such hoaxes created? One writes a text that sounds serious, scientific, and well-researched, but plants there several treacherous claims (*traps*). They are likely to be missed by unsuspecting referees and editors-in-chief, but the absurdity of these claims can later be readily disclosed to compromise those referees and editors.

So, let’s look at the paper and find the traps before it’s too late. One of them is based on what I call the “method of Chelmites”, referring to one of the anecdotes (I learned it from Berkeley storyteller Joel ben Izzy) about Chelm, the folkloric Jewish town of fools: To protect their town from rain, Chelmites first build a wall around it, and when all their treasury is gone but the rain is still pouring, they solve both problems by renaming water *money* and money *water*. The authors of the hoax paper virtually quote this story when (on page 10) they

... advocate for the redefinition of *prestige* and *elitism* in mathematics in a way that better reflects equity for excellence. For example, the institutions that might deserve our respect are institutions that reflect the diversity

of the US population among their faculty and doctoral graduates in mathematics.

They proceed by presenting 10 most *prestigious* (in Chelmites' metric) PhD-granting mathematics departments, knowing pretty well that research journals don't fact-check their submissions. But we will.

The closest to math doctoral program I could find at *Drew University* was in theology. The highest math degrees offered by *Cleveland State University* and *the University of Texas at Tyler* are Masters. *Illinois State University* and *Teachers College Columbia* offer PhD degrees not in Math but in Math Education (which, as Russians say, are 'two big differences'), while *Louisiana Tech* lists some in "Computational Analysis and Modeling" but none in math per se. *University of New Hampshire* currently homes 1 math PhD student (specializing in analysis). The remaining three deserve more attention.

The youngest of UC campuses, *UC Merced*, apparently has a strong research group in numerical methods, and judging by their list of [recent graduates](#) produces 3 math PhDs a year. *Case Western Reserve University* (in a basement of which the celebrated Michelson–Morley experiment was once performed) has a joint math and stat department, currently with 37 Masters and PhD students in total, and so probably awards 2-4 PhD degrees in math yearly. Finally, *Bryn Mawr*, the women's liberal art college where Emmy Noether spent her last two years, has a small math department which on average produces one PhD a year. Thus, in the city of Chelm, the math faculty of the most prestigious ten produce PhDs at a rate barely sufficient for their own timely retirement.

And what about *elitism*? It turns out that even *outside* the city wall of Chelm avoiding traps is a challenging task. There a department is *defined* to be prestigious if its graduates are hired by prestigious departments (i.e. the authors pretend that they've never heard of circular reasoning). The main charge of elitism goes to the NSF's Department of Mathematical Sciences (DMS program) whose whopping 86 percent of funding goes to 20 percent most "elite" (= prestigious?) departments. Here the authors invoke the Pareto 80/20 principle in economics according to which the most successful 20% normally absorb 80% (rather than 86%) of all resources. This scientifically sounding discussion serves only one purpose — to distract the reader from the trivial question about *weights*: Is there a chance that those elite 20% of all departments account for 86% of all math faculty? PhDs? new theorems? While referees and editors are thinking whether this is possible, let me use this short break for a bit of self-advertising.

My own math department at UC Berkeley currently houses some 190 PhD students, and awards about 30 PhD degrees each year. The department is truly diverse (in the original, non-euphemistic sense of the word), with faculty (of whom many are members of various academies and recipients of national and international awards) working in virtually all branches of mathematics: algebra, geometry, topology, number theory, analysis, applied math, probability, logic, you name it. Some hold joint appointments at and tight connections with physics, statistics, computer science departments, and with the Lawrence Berkeley National Laboratory. Besides, the SLMath (a.k.a MSRI) hosts hundreds of visitors working in currently most active research areas. Some seminars are held jointly with Stanford and UC Davis which are within an hour-long drive. Thus, this is one of the most vibrant mathematical communities on the planet, and so it is not surprising that it attracts brightest graduate students and postdocs. What is discussed in our graduate courses, research seminars, and colloquiums lies at the very frontiers between modern and future math. Our course catalog is long, and our best undergrads finish with an impressive array of graduate level courses in their portfolio, which allows them to go on to the most competitive PhD programs. Our outreach includes running the Bay Area Mathematical Olympiad and the Berkeley Math Circle (BMC) for middle and high school students, while our BMC Elementary program is attended by hundreds of kids in grades 1–4. Our BMC summer camps over Zoom bring together students and instructors located all over the Globe, and the educational materials produced by the BMC are accessible to all. So, the place is elite based on merit.

Returning from the break, let me note that the mock charge with the unfair distribution of NSF funds is only one of two pillars of the hoax paper. The other mockery, concerned with under-representation of women, is disguised even more subtly.

On page 2 it is explained that within the last decade women are awarded 30% of PhD degrees in mathematics. On page 6, however, the orange graph shows that the fraction of women in the math faculty during this decade *increased* from about 14 to 18 percent. So, the authors hope that the editors and referees won't add 2 and 2 together, but we will: if the trend continues for another 30 years, i.e. for the entire 40 year period of a typical academic career, the figure will reach the requisite 30%. More precisely, the authors know that the gender statistics of faculty hired *in the past* can't reflect the *current* fraction of women receiving PhD in math when the latter is *growing*. What is at work here is a BMC Elementary-level theorem which says that *the average can equal the maximum only when the function is constant*.

To illustrate the point: in my home department’s current list of 100 [Senate faculty](#) 13 are women (and I believe this 13% value was entered by the authors for UC Berkeley in their charts). However, out of the 100 total, the 40 who have already retired are all men (for Marina Ratner passed away in 2017) and so the updated fraction is over 20%. Yet, among Assistant and Associate Professors, it is $4/13 > 30\%$.

Thus, saying (on page 2)

...if mathematics were indeed a meritocracy, on average,
30% of faculty at mathematical sciences PhD-granting
institutions would be women,

the authors know they are not being honest. But as if to make sure that the reader doesn’t escape the snare, they plant one more trap:

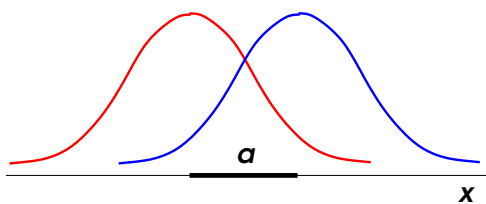
We assume... Therefore mathematics is not a meritocracy.

As everyone knows, “if A then B” doesn’t make B true until A is verified. And the authors know that what they assumed as A — that in meritocracy, women would make the same fraction (30% in this case, they claim) in *every tier* of the achievement ladder — is factually false. But this time, I think, they honestly don’t understand why, and they hope that we don’t understand either. So, let’s help ourselves.

The higher attrition rate up the ladder could be due to bias. Yet, there are many examples where it is hard to suspect any non-meritorious forces in play. One is chess: among [FIDE rated players](#) 11% are women though among grandmasters they make $< 3\%$. The distributions of men and women by FIDE rating look not too far from Gaussian, with almost identical *spreads* but different *means*. Another random example: [a study](#) shows that in Germany’s several-tier system of selection of high-schoolers to the International Chemical Olympiad, the portion of girls decreased with each tier (in 2017 from 48% to 0), and a similar decrease is found among the recipients of awards (honorable mention, bronze, silver, gold) at the Olympiads themselves.

We must leave it to experts (rather than hoaxers) to investigate any specific causes, but let us consider the following simple cause-independent model. Imagine a school whose students are interested in two (for the sake of simplicity) equally popular subjects (let’s call them **B**iology and **G**eography), and where for whatever reason boys on average are more interested in the former, and girls in the latter. Consider now the score distribution on a test administered to all students, which measures (not raw ability but) proficiency in Biology, and another such test in Geography. Assuming the score distributions for

boys and for girls are both Gaussian — something which is not guaranteed, but is backed up by the Central Limit Theorem in probability theory — and (to avoid the topic which costed their jobs to some and their nerves to many) that they are Gaussian *with the same spread*, we find that the otherwise identical distributions must have *different means*: $e^{-x^2/2}$ for boys and $e^{-(x+a)^2/2}$ for girls, where $a > 0$ for the Biology test (simply because girls on average were assumed to be less interested in it) and $a < 0$ in the other test, for symmetric reasons. The ratio Ce^{-ax} of these distributions (where $C = e^{-a^2/2}$) is the ratio of girls to boys in each particular test score x . We see that this ratio does not stay constant — contrary to what the authors of the paper *assumed* — and in the Biology case ($a > 0$) decays exponentially with the increase of x : the higher the achievement tier, the lower is the share of girls expected in it.



The above model doesn’t have to be always true, but it is basic enough to illustrate the fact that the assumption, made in the hoax paper, that in pure meritocracy the achievement distribution of women which go into math must be always proportional to that of men, is ungrounded. Moreover, in the cohort of US high school entrants, the percentile of future PhD recipients (in all subjects) is currently at about the level of 3 standard deviations. Within this percentile, the ratio 3/7 of women to men *corresponds* to the difference a between the means in our model of about 0.25 standard deviations, which is at the scale found in math **ACT** and SAT scores.

The assumption in the hoax paper was made not because it was provably correct, but because the opposite sounds politically incorrect. It so happened that when I first saw the paper, I was reading *Social Justice Fallacies* by Thomas Sowell (Basic Books, 2023). In this work, Sowell provides multiple factual counter-examples to various modes of ideologically correct reasoning. It appears that several of these modes were intentionally employed by the authors of the paper.

Sowell’s overarching observation in the book is that the progressive politicians and ideologues promoting the social justice agenda at the start of the 21st century agree with the progressives of the early 20th century era in most aspects of political discourse (e.g. the role and

size of government) except one: The outcome disparities, which are now blamed on racism and sexism of the society, a century ago were assumed — by the like-minded intellectuals! — to be the result of racial and gender *determinism*, a stance which to our modern eye is clear-cut bigotry and Nazism, but which contradicted (as Sowell shows) to many historical facts lying in plain view even then.

Sowell is generous to the intellectuals of both eras when he refuses to suspect them in being insincere, but he hints briefly at the tragic history of the last century, when the regimes espousing the social justice ideology evolved invariably into bloody totalitarian dictatorships. In fact, those who have a lived experience of such regimes know that the mechanism of this transformation has little to do with any specifics of the ideology.

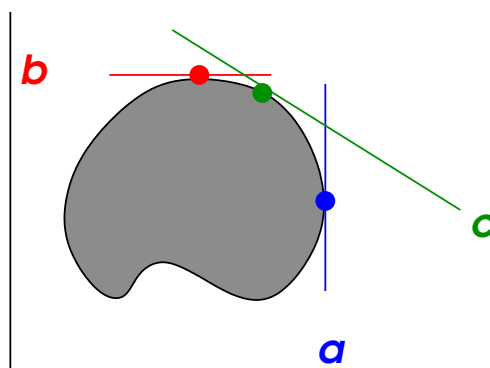
It is very simple. In meritocracy, upward mobility is driven by talent, skill, expertise, perseverance — something that is hard to fake. But the allegiance to an ideology (and it doesn't matter to which one) is easy to fake, and is virtually impossible to distinguish from genuine zeal. Once this allegiance becomes a means of upward mobility, cynics enter and win the competition.

The phenomenon is easy to illustrate with the matter at hand. There is no doubt that the authors of the paper planned a hoax exposing confirmation bias in the trade. But if their “traps” were less obvious, how would one distinguish them from honest misconceptions, or worse — from blunt exploitation of political correctness for personal gain? In the current ideological climate, a research paper asserting that gender and race disproportions in the US math departments are not due to any biases, and that NSF funds are distributed strictly on merit, would have little chance to be published — and hence written! — which even without any foul play guarantees the confirmation bias in the field, and is in itself an obstruction to meritocracy.

Unfortunately not the only one. We have shown above how simple mathematical models help dispelling some ideologically correct misconceptions. Let's apply this approach a few more times.

I once worked on the committee awarding the University Medal to the best undergraduate of the year, a title defined in terms of five different criteria. What members of such committees seldom realize is that it is impossible to optimize on more than one criterion. The reason is simple: to optimize, one needs to decide which of any two — in this case students — is better. While points on the number line are naturally ordered, on the plane (with two coordinates a and b) they are not. Optimizing for one of them (on a set of points bounded by outer constraints) can only be achieved at some expense to the

other (and if accidentally both maximize simultaneously, then it doesn't matter which one to use). One can choose to optimize some *weighted* combination, say $2a + 3b$, but this is yet another quantity c , and its optimum can only be achieved at some expense to both a and b .



For example, numerous education boards are concerned with (a) improving education, and (b) eliminating various achievement gaps. In fact there is an easy way to solve (b): prohibit learning anything at all, and the gap disappears! To a mathematician, such unrealistic extrapolations show the main tendencies. In fact this tendency is materialized in what the California Board of Education is currently doing by re-designing the state's Mathematics Framework: it purports to lower the achievement gap of minorities by limiting access to advanced math education for all! Those honestly misled by ideologically correct phraseology have a chance to be convinced by mathematicians' arguments, but what are the society's safeguards against those cynics who simply exploit the ideological climate to their personal advantage?

Although, as I said, it is not possible to order things by more than one parameter, we regularly do so and to a great success — for instance, when we use a dictionary. That's why mathematicians call such ordering on the plane *lexico-graphical*: first compare the coordinate a , but when equal, use b . In real life this method is commonly known as *prioritytizing*. This is the only ethical way to influence the demographics of, say, graduate admissions: Admit all who are clearly above the cut-off group based on their merit, and within that group apply a demographic criterion of your choice.

As far as I understand, a different approach is taken in the University's hiring process. For example, our department's Faculty Appointment Committee (FAC), after a long and elaborate process of examining hundreds of job applications, compiles a short list and passes it up to the college's Office for Faculty Equity and Welfare (OFEW). In

2019, the latter *decided to bar from our search all tenured applicants, as that sub-pool of applicants did not meet their demographic standards.*

We leave it for game theorists to analyze this two-player game between FAC and OFEW, of which one is to be primarily concerned with professional quality, and the other with demographic diversity. One thing is clear though. While there is no doubt that any candidates from FAC's short lists, if hired, would become spectacular additions to our department, even in this distinguished group of equals some could be more equal than others. Consequently, in 2019, OFEW's policy could negatively affect the department's professional quality. What is at work here is a simple mathematical theorem: *The maximum over a sub-pool cannot exceed the maximum over the entire pool.*

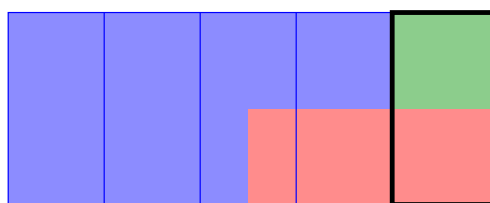
Furthermore, to those candidates who are (using the newspeak) not diverse enough, a back door is offered. Namely, job applicants are expected these days to submit a statement explaining how their actions are going to promote diversity in their profession. Some, especially those who have a lived experience of ideological totalitarian regimes, might consider this as bowing to an ideology, no matter how attractive, and won't submit it (which is hardly an option for post-doc and tenure-track applicants). But others might view it as a welcome opportunity to get ahead by means which require neither talent nor expertise. Thus, we must concede the point (incorrectly justified in the hoax paper) that nowadays hiring at elite math departments is not based on merit only.

Our last application of basic mathematical reasoning in the matters of social justice is best illustrated in the context of racial discrimination. There can be any number N of adversities in people's life. One can be poor, have poor health, have no medical insurance, no education, no job, no profession, be a drug addict, homeless, have a parent in jail, etc. Given the troubled American history of slavery, racial segregation, and discrimination, it is not hard to believe that each of these conditions correlates positively with being black. Yet, no two groups affected by those adversities coincide. According to the data presented by [Tricia Rose](#) (thanks again to the BLM reading group), over 17% of Blacks considered in 2009 that racial equality in the US had been achieved, i.e. even the group of Americans who subjectively experience any racial discrimination is not identified by race. While denouncing *color-blindness* as a form of racism (as Rose does) is a good strategy for virtue-signaling in Chelm, the question remains whether *color-consciousness* is a good strategy for policy-making outside of it.

A mathematician's approach is to start with the simplest case $N = 1$. Imagine an adversity which disproportionately affects African Americans, e.g. 50% of them while affecting only 25% of general population.

(In Rose’s lecture it was [city redlining](#) of the 1930-ies, where the data about the black population of type D districts varies from $< 15\%$ to $> 60\%$ depending on the city and the source of information.)

A color-conscious policy would attempt to alleviate the adversity by targeting the most affected racial group (say, by issuing interest-free home loans to African Americans in the above example). Assuming (for the sake of the argument) that 20% (outlined in black on the picture) of urban population are black, we find that: (a) of the population affected by the adversity (red), 40% are adequately compensated (because they are black), but 60% remain uncompensated (because they are not black), and besides (b) 10% of the entire population (green) is unduly compensated (because they are black not affected by the adversity), which makes all the remaining 90% disadvantaged by the policy in comparison with those 10% privileged by it.



All this mess is avoided if the policy is informed by color-blindness, i.e. when the alleviating measure is applied to the members of the affected group only — regardless of their race. We leave it as an exercise for the reader to examine the case $N = 2$ (or any other N) where color-consciousness leads to even worse results while color-blindness solves the problem perfectly even when the effects of different adversities compound each other.

However ideologically incorrect the above conclusion sounds, the fundamental reason behind it is simple: *There is nothing inherently wrong with being blue-eyed — or black-skinned.* Yet, the ideologically correct view implicitly assumes the opposite, i.e. turns out to be identical to the century-old bigotry of “racial determinism”.

P.S. I am finishing this essay being already expelled from the BLM reading group for *misfit* — resorting to reasoning, referring to data, raising questions (and paying little respect to postmodernism). Coincidentally, the group (which therefore practices DEI interpreted as Discrimination, Exclusion, and Indoctrination) is funded by an NSF grant in mathematics. Thus, we have to concede the other point too: nowadays the distribution of DMS funds is not based on merit only.