Algebra for All Ages

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For most of the twentieth century, the study of algebra was reserved for those students who had completed courses in arithmetic and were entering high school. In the 1980s, the College Board commissioned a study to examine the relationship between students' high school course-taking histories and their successful entrance and completion of college. The findings showed that students who had completed Algebra 1 were most likely to enter and complete four years of college. In response to this study, school systems nationwide began requiring all students to enroll in Algebra 1, and to do that no later than grade 9. As one might expect, great numbers of students failed Algebra 1; they had severe difficulty making the transition from programs requiring arithmetic thinking to courses requiring abstract algebraic thinking. The solution to this problem was to provide a continuum of instruction in algebra, beginning in the early elementary years and continuing until entrance into formal courses in algebra at the high school level. In the *Curriculum and Evaluation Standards for School Mathematics* (1989), the National Council of Teachers of Mathematics (NCTM) identified algebra as a major strand of the curriculum for students in grades K–12. In 2000, in their *Principles and Standards for School Mathematics*, NCTM reiterated the need for the study of algebra at all levels of instruction.

WHAT IS ALGEBRA?

Algebra is sometimes referred to as generalized arithmetic because it formalizes arithmetic relationships. Its power lies in the fact that it offers ways to represent relationships among quantities, to describe properties of operations with sets of numbers, and to describe patterns. Algebra provides rules for manipulating symbols, such as simplifying an expression and then solving for an unknown.

In the past, instruction in algebra focused exclusively on the manipulation of symbols and the solution of equations. Now, instruction focuses more on the development of certain key concepts or big ideas.

THE BIG IDEAS OF ALGEBRA

Variables and Equations A variable can be an object, a geometric shape, or a letter that represents a number of things. Variables are used in three ways in elementary school mathematics. They are used to represent unknowns, to represent quantities that vary, and to generalize properties.

Represent Unknowns

An *unknown* is a variable that has a fixed value. Variables with fixed values are used extensively in equations in the early grades, for example, $5 + 4 = n$ and $s + 2 = 9$. As students progress through the school years, they learn to solve systems of equations with two unknowns. They learn that the same letters or shapes in equations represent the same values. They may solve problems like this.

$$\square + \Delta = 10$$

$$\square - \Delta = 2$$

Word problems pose an additional challenge. Students must represent or write an equation to show the mathematical relationship(s) and then solve it. Consider this problem, which is structurally like the square-triangle problem above.

> The wingspan of the butterfly is 2 cm longer than that of the moth. The two lengths are 62 cm. What is the wingspan of the butterfly? What is the wingspan of the moth?
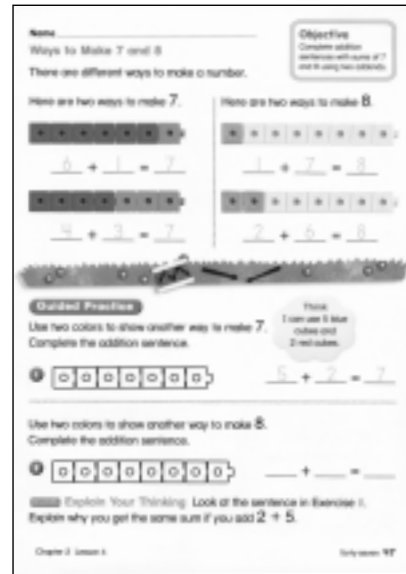>
> From *Houghton Mifflin Math*, grade 3 Student Book, page 90.

If b stands for the wingspan of the butterfly and m stands for the wingspan of the moth, then $b - m = 2$ and $b + m = 62$ cm.

With effective instruction, students learn to identify different representations of the same type of problem so that similar solution strategies can be applied.

Represent Quantities That Vary

In some types of equations, the variables are not fixed; that is, they may assume different values. Students are introduced to the concept of variables as varying quantities in this example showing different ways to make 7 and 8.

Student Book, grade 1, page 47

In this primary level example, from *Houghton Mifflin Math*, two line segments are used to stand for the pairs of addends that sum to 7 and 8; the first and second numbers in the pairs may vary.

In later grades, students solve the same type of problem when it is shown as $a + b = 7$ or $a + b = 8$, and a and b are designated as whole numbers.

Variables that take on different values are found in all formulas. For example, the formula for the perimeter of a rectangle is $P = 2l + 2w$, where l and w may be different for different rectangles. The formula for the area of a triangle is $A = 1/2\,bh$, where b and h vary for different triangles. Likewise, in the distance formula, $D = rt$, the rate and time will vary for different situations.

Generalize Properties

Early in their mathematics education, students are introduced to special relationships that exist among the elements of sets of numbers under specific operations. Understanding and applying these relationships reduces the number of facts that

students need to learn and can simplify computations. For example, if students learn that $4 + 1 = 5$, then they also know that $1 + 4 = 5$; they do not need to learn the second as a separate fact. This is an example of the Commutative Property of Addition of whole numbers. It applies to all pairs of numbers in the set of whole numbers, and can be represented as $a + b = b + a$. In the primary grades, students' participation in activities such as this help them develop algebraic thinking and understand the properties of operations.
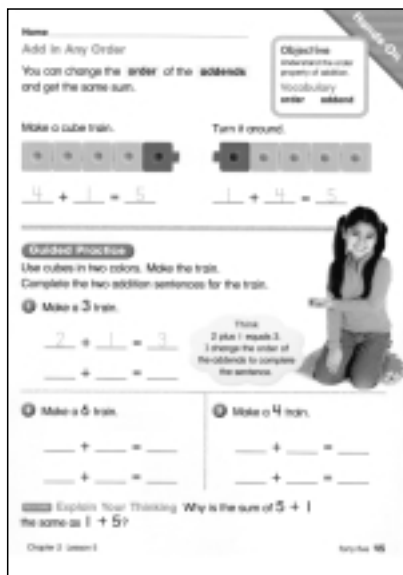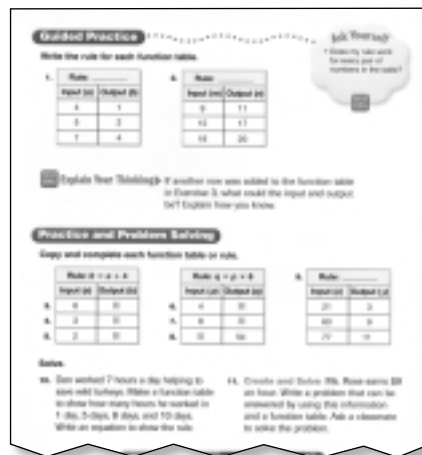
Student Book, grade 1, page 45

Likewise, $a \times 1 = a$ is the Identity Property of Multiplication. It states that when any whole number is multiplied by 1, the product is equal to the whole number. Using variables provides a shorthand way of describing relationships that apply to all numbers in a set.

Patterns and Functions In the early grades, students are exposed to two different types of patterns: those that repeat and those that grow or shrink. A repeating pattern is one in which a group of elements repeats, for example, AABAABAABAAB. A growing or shrinking pattern is one in which every element in the pattern is related to the preceding element in the pattern in the same way. An example of a growing pattern is 0, 2, 4, 6, 8, 10, 12,…; a shrinking pattern is 100, 90, 80, 70, 60,…. It is growing and shrinking patterns that lead to generalizations and to representations of the generalizations using variables.

Children learn to describe growing and shrinking patterns in words and to predict what comes next. So they might say, for the growing pattern above, that "each new number is 2 more than the number before it." In the intermediate grades, students learn to describe this pattern by writing $y = n + 2$, in which y represents the new number and n represents the number before it.

Student Book, grade 4, page 127

Students identify patterns when they complete function tables. They learn that for every input there is exactly one output. In the early grades, students complete function tables when the rule is given, for example, *Add 2*. In the later grades, students not only complete function tables when given the rule, but in some problems they also find and record the rule, as shown here.

By grade 5, students are identifying patterns, collecting data, constructing tables to display the data, and generalizing the patterns by writing functions with variables, as exemplified here.
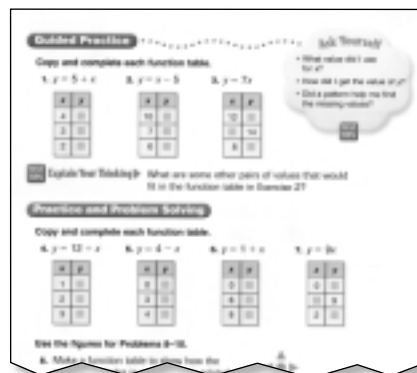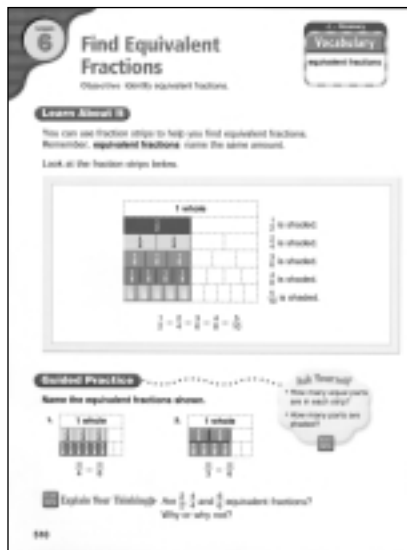
Student Book, grade 5, page 577

PROPORTIONS AND PROPORTIONAL REASONING

A proportion is a special type of function. It is a relationship between quantities that is multiplicative in nature. Reasoning about relationships that are related by multiplication is referred to as proportional reasoning. In elementary and middle grades mathematics, proportional reasoning occurs everywhere! In the study of place value, students learn to regroup tens for ones; 1 ten is equal to 10 ones, so 2 tens is equal to 2 x 10 or 20 ones, 3 tens is equal to 3 x 10 or 30 ones—all regrouping involves multiplication. In the study of measurement, when students reason that there are 4 cups in one quart, 8 cups in 2 quarts, and 12 cups in 3 quarts, they are reasoning proportionally. When they trade 5 pennies for 1 nickel, 10 pennies for 2 nickels, and 15 pennies for 3 nickels, they are again using proportional reasoning. When students construct equivalent fractions, as shown in this example, they apply proportional reasoning.
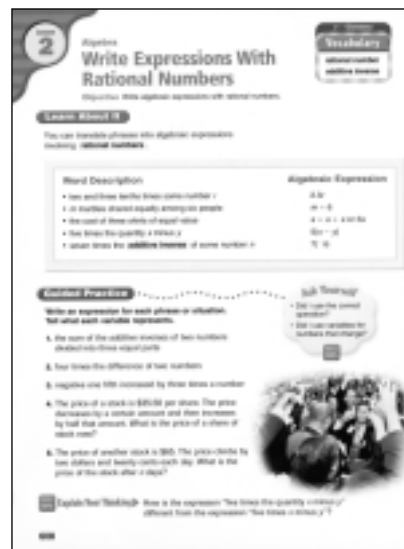

Student Book, grade 3, page 510

In the late elementary and middle grade levels, students learn to record proportional relationships by using symbols. They also learn to graph the relationships.

The graph of a proportion is a straight line that contains the origin. A strong foundation in proportions and proportional reasoning sets the stage for the exploration of other types of functions at the high school level, functions whose graphs do not contain the origin and may not be straight lines.

THE ROLE OF THE MATHEMATICS TEACHER

As this paper explains, there is a strong connection between arithmetic and algebra that is important for teachers to emphasize to their students. When teachers come across problems of the types described here, it is necessary to spend time discussing and solving them. Teachers can also find other problems like these and give them to students, or show students different representations of the same problem. Students can then describe their thinking, document the steps they followed to solve the problem, and provide a rationale for their solution approaches. In this way, students learn different approaches to problems of the same type and are guided to explain their thinking.

Student Book, grade 6, page 608

Developing students' algebraic thinking, beginning as early as the kindergarten level, will enhance their understanding of the concepts of number, place value, measurement, money, and fractions, to name a few. Moreover, this development prepares students for future mathematics learning in algebra courses and beyond.

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