How the incompetent enlightens the ignorant, ***by Alexander Borisovich***

About a decade ago, "Primary Math", a program for grades 1-6, as well as its companion math program for Kindergarten, were imported from Singapore to the US soil. They are indeed excellent programs, correct mathematically, and can be used in a simple and very efficient way for bringing kids to mastery of basic math. I was using it with my son, and can report that not only it worked for him, but being thereby forced to think, I myself have learned to understand a myriad of mathematical subtleties which a first-time learner needs to absorb, and which all become indiscernible from a more advanced standpoint.

However, the history of Singapore math programs in the US actually leads to the following sad conclusion: For bringing the basic math education in the US up from that deep hole in which it is lying, neither Singapore math, nor anything else would help. Ignorance is invincible – judge for yourself.

Actually it started quite promising. The program became very popular among home-schooling parents who talked of great success, and also of the joy and pride with which kids could often learn new lessons from Singapore books without any adult's supervision. Arrangements for the US edition of "Primary Math" were made. Two mathematicians (T. Parker and S. Baldridge) wrote a textbook for pre-service teachers based on this program. Some other mathematicians ran pilot programs in schools, and organized professional development sessions using this program. Later "California Edition" was designed and passed the approval of the state's Board of Education. Interim, a 200-page comparative study of Singapore and US math education systems was published. There has been no shortage in the news media exposure of Singapore Math as well.

In retrospect, the first signs of the disaster were not too hard to notice, as they can even be tracked back to the textbook of Parker and Baldridge. This is a highly praised and very well written book. Unfortunately, it essentially begins with grade 3 (as if there was nothing important in math of the earlier grades), and spells out only in general words the advantages of the Singapore Math program. When it comes to "word problems", the book explains, quite correctly and efficiently, I think, the essence of that neat way in which the Singapore textbooks present solutions: by representing quantities by line segments, the same way as it was done in Euclid's "Elements." I felt however that the attention to these "bar models" was somewhat hyper-inflated. In any case, as it comes to the issue why word problems are there at all, the authors only resort to a joke, leaving the US public in the same woods as it has been on this issue for decades.

At the same time, some professional development workshops in MA run by a mathematician with the aid by former school teachers – participants of first pilot programs, began to train more teachers to use Singapore Math in school, and – internet discussion boards in MA became filled with one primary issue: "bar models." Soon those former teachers-instructors spread their own wings and began to run professional development workshops in other states. At about the same time, they started giving interviews in newspapers, and in other media, emphasizing "many remarkable features" of Singapore Math programs. But when it would come to any specifics, invariably, only one and single point would be made: Singapore "bar models". It looked like this was the magic wand that was going to save America from its total innumeracy.

One day, in the classroom of my son, I discovered a poster copyrighted by Bob Hogan, one of those self-appointed teachers-experts on "Singapore Math." According to a news interview, the former 5th grade teacher was already working on his PhD in education, and the poster apparently showed his

main scientific discovery: An 8-step plan, which kids needed to memorize and then execute each time they attempted to solve a word problem. Here is the plan:
Read the entire problem *(I think the poster said:* twice) - decide who is involved - decide what is involved - draw unit bars of equal length (?) - read each sentence one at a time - put the question mark in place - work computation to the side or underneath - answer the question in a complete sentence.

I thought it was obvious that this plan has just as much to do with solving math problems as the advice to sharpen your pencil and record each move – with solving chess puzzles. I was wrong. The scientist himself soon visited the school, and the Principal sent emails to all parents announcing that now we finally know why Singapore Math is good, and that all parents are requested to help their kids to memorize the steps.

The Principal was soon fired by the Board of Trustees, and went to another school, where he also implemented Singapore Math. By then, the program became so popular, that teachers from many schools formed a community where they were teaching each other the "Singapore method" of solving every problem …in 8 steps.

The poster was only a beginning. Soon Bob Hogan, together with another expert, Char Forsten, published a book: "8-step Model Drawing: Singapore's best problem-solving math strategies". It is a guide for teachers how to teach students to execute the 8 steps: what the teachers should draw on the board, and which words to say.

Recently, it turned out that Char Forsten published her own guide: "Step-by-step model drawing", intended "for struggling students". Trying to argue on the Amazon.com page with the book's proponent, I was assured that it is very different from the previous one. And indeed, it turned out that Hogan's plan was incorrect, and in the correct one, Step 2 should be: to write the answer sentence with a blank in it. *The proponent told me that she achieves better results using Forsten's way, and that other high-school (?!) teachers who she teaches the method said that if they knew that this should be Step 2, their students would have had a completely different year, for they would have been able to focus on the question they were asked. (I'd be curious to know if those high-school teachers, before going to classes, smoke the same stuff that their students smoke.)*

*Meanwhile, another book in the same series was published, called "*Model Drawing for Challenging Word Problems: Finding Solutions the Singapore Way, Grades 6-9" (?). *The sample pages representing this book at SingaporeMath.com bookstore website are the same as for the previous one, for "struggling" students of grades 1-6. The demand for this literature is great.*

Youtube is looming with demonstrations of how to teach math "the Singapore way," most of which are about drawing "bar models" no matter what. Nobody is ever looking into Singapore math books anymore to find out how indeed they teach particular topics, in what order, in which grade – everyone like crazy is drawing bars (I don't know why I wrote: "like"). Math education blogs are also filled with self-appointed experts' disclaimers of their personal histories of falling in love with "Singapore's Method". That is how it usually goes.

Someone who did not learn in primary school how to solve elementary math exercises, but learned in high school how to do them with algebra, becomes a college failure, hence an English or education major, sympathizing with those who, like him, experience difficulty learning, and thus eventually ends up teaching math in grade 8 . Then he discovers that first grade teachers in Singapore know math better than him, and says: Aha! It turns out "there is research showing that teachers' subject *knowledge affects the quality of their teaching" (I am not kidding – I am quoting). So, he – finally! - learns from Singapore books how to solve 5th grade exercises without algebra and – guess what? – begins to educate others on the virtues of labeling bar diagrams, in 8 steps, or in his own ways, to give interviews, lead blogs, pose on youtube, tv, etc.*

The incompetent enlightens the ignorant, and the infection spreads faster than plague.

I append below the review I posted to Amazon.com based on the sample pages of those two books on "model drawing." The review is linked to the samples themselves, and so is not really needed. If you don't see from the samples how crazy they are, it means that you are already infected, and my review is not likely to cure you.

Diagnosis: Incompetence, ***by Alexander Borisovich***

I have previously written briefer negative reviews of two similar books on "step-by-step model drawing" - one by Char Forsten, another by her together with Bob Hogan, and recently had a discussion on the Amazon page about the first of them with Susan Midlarsky who, based on her years of experience of teaching by such books, is very fond of them. If you can see right from the covers of these books why they are not only useless but rather dangerous, you don't need this review. If in the contrary you are a teacher or parent thinking that these books might help you, it probably indicates that you have little confidence in basic math and/or in your ability to teach it. In this case, my review probably won't convince you. Nevertheless, it does not hurt me to tell you: Ignore this literature; buy the whole set of Singapore Math books for grades K through 6, learn how to read them (it's easy - kids can do this), read them, understand all the "why"s, solve all exercises asking yourself constantly this question; and when you learn basic math this way, you'll be able to teach your kids or students the way *you* learned it. Believe me: Singapore books themselves contain all one needs to study from them. The book under review don't; they are outrageously incompetent, and exist only because there is a demand from buyers like you, who fear math and cannot distinguish what's true about it from what's false.

The rest of this review contains a thorough analysis of the 4 sample pages which are displayed at website singaporemath.com for both this book and its twin by Forsten. I don't actually know from which of the books the sample pages are taken (nor why this particular book is labeled as intended for grades 6-9: Does it mean that US students are supposed to wear in the 9th grade the clothes of which Singapore students grow out in grade 7?) Anyway, my goal is to show why exactly these samples demonstrate outrageous incompetence of the books' authors. Brace yourself for a long ride.

Two of the sample pages are found at
http://www.singaporemath.com/v/vspfiles/assets/images/sp_mdswpsw1.gif
http://www.singaporemath.com/v/vspfiles/assets/images/sp_mdswpsw2.gif
They represent an example of how a teacher should guide her students to solving a word problem following an 8-step plan invented by the authors of the books or their colleagues.
Only 5 of the 8 steps are explained on these pages, the remaining 3 are apparently explained on the next page, not displayed among the samples. The left half of each page contains descriptions of the steps and pictures with "bar models", the right halves contain the text (script) that a teacher is supposed to tell to the class (thus, totally 1.5 pages of the text, I assume).

The actual problem in question is: "Rick has $45. Tom has $28. How much more money does Rick have than Tom?"

One should be totally insane in order to recommend teachers to try this 1.5-long script and the tedious 8-step procedure on live children, for this is a sure way to convert them into intellectual cripples and divert them from mathematics.

Exercises like this occur in Singapore Math (=SM for short) at the beginning of grade 2. There are no 8-step procedures in SM, there are no 1.5-page long scripts. The students of grade 2 are expected to immediately figure out that in order to answer the question, one needs to subtract $28 from $45 and thereby to find the answer: $17. As an aid for those who has not figured it out, there will be a picture in SM, showing two "bars" (one for Rick's, another for Tom's money, with the question mark against the portion of the longer bar that sticks out. The picture is meant to be self-explanatory.

The authors of the books under review will tell you that not all children can understand the picture, and that's why their 1.5-page long script is useful. Don't believe them: their script is incompetent and has nothing to do with the actual solution of the exercise. A teacher that would like to explain her way of reasoning would do something like this. She would draw a line on the board saying (let me give you my script): "This is Rick's money; How much should I write here?" Some of the children would look in the problem and say: 45. Then draw another line, shorter, for Tom's money, and the children would say: 28. She might ask: "Why did I draw this line shorter?" and someone would answer:
Because Rick has more money. Then she would ask: By how much more? probably putting a question mark next to the part of the longer segment that sticks out. The children would figure out that it is the difference, $45-$28, and then do the computation. This is all it takes to explain the solution: it is quick and straightforward. All of what you find on the sample pages is irrelevant.

The authors of the reviewed books will argue that they have the experience: many children won't be able to understand this discussion, that they need the solution to be chopped into many steps, need to read and re-read the formulation of the problem, do it one sentence at a time, etc. Do not trust them - they are incompetent. They will tell you that word problems are hard because they contain words, and 2nd graders have difficult time understanding the words. Don't buy it: The problem is stated concisely and clearly. The same children, when it comes to baseball, will talk in incomplete sentences 5 times faster that you can and understand each other perfectly. Word problems are hard not because they contain words, but because they require understanding of mathematics. To help the children who have difficulty with a problem, the teacher needs to understand what mathematics is needed in each problem. The authors of the book do not.

For this particular exercise, one needs only to understand the connection between the binary relation "more than" and the arithmetic operation of subtraction. Thus, if a student struggles, it means that he either does not know the meaning of "more", or the meaning of subtraction, or both. To help the student, there is only one way: to teach him, by returning to the "concrete" (as opposed to pictorial) manipulating with these notions. The notion of "more" is taught in SM at the beginning of Kindergarten program, the meaning of subtraction at the end of it (if I remember correctly) and in grade 1. Use that material. Make the student compare lengths, weights, heights; quantities of beads, pencils; ages of their siblings, etc. When the meaning and relation of "more" and "subtract" is firmly abstracted in the student's minds from these concrete manifestations, they'll be able to solve the exercise at hands on the flight.

Nothing on the sample pages in this book goes toward this goal. The script is talking about names of boys (irrelevant!), two variables (the term that does not occur in the elementary SM program at all), unit bars (there is no such a term in SM, nor such a notion in mathematics at all - and I really have no clue why the authors call them "unit"); about drawing the bars first equal, then adjusting the lengths (assuming thereby that students do know what "less" and "more" mean). All of that is outright incompetent gibberish. All who I know, at seeing this 8-step plan - even without the script - only twist their forefingers at their temples: "Are they out of their mind?!" But in some parallel universe called "US Math Education" these people are considered "experts" teaching others how to teach our children! (At this point, the authors will tell you: maybe *my* children don't need this, but there are many "struggling" children who do. Don't believe them: *your* children also don't need this; they need math, plain and simple.

Let's go to the next sample page
http://www.singaporemath.com/v/vspfiles/assets/images/sp_mdswpsw3.gif
It contains 5 exercises on fractions for your independent practice. It shines with samples of the author's incompetence.

The first exercise is: "A pizza is cut into 5 slices. Fred ate 3/5 of them. How many slices were left?"
In fact the problem is incorrectly marked as a problem on "Subtraction" of fractions. It is an exercise on understanding the *definition* of fractions. By this definition, 3/5 means 3 out of 5 equal parts. Since the pizza is cut into 5 slices, it means that 3/5 of them is 3 slices, thus 5-3=2 slices are left (so the subtraction of whole numbers, not fractions is needed).

The next problem is incorrectly labeled "Multiplication" of fractions: "Kim found 6 plates of pizza left after a party, with 1/2 of pizza on each plate. She put the halves together. How many whole pizzas did she get?"
Since 2 halves put together make a whole pizza, then the total number of whole pizzas is 6/2=3 (thus obtained by division of integers). You can also say that 6 x (1/2) = 6/2=3, i.e. use multiplication. You can also argue this way:
6 halves are obtained by taking a half of each of 6 whole pizzas, and a half of 6 is 3. Actually the equivalence of these three ways of solving the problem is the non-trivial mathematical fact about it, and deserves thinking, but requiring that this problem be solved by multiplication betrays incompetence of the authors.

The next problem is incorrectly labeled as "Division" of fractions: "Four people shared 1/2 of a chocolate bar equally. How much did each of them get?" A possible solution: since each of them got 1/4 of 1/2, and 1/4 of whatever is the product of 1/4 and that whatever, the answer 1/4 x 1/2 = 1/8 of the bar, i.e. is obtained by multiplication of fractions. Of course, you can say that sharing equally means dividing 1/2 by 4. The fact that dividing by 4 is equivalent to multiplying by 1/4: this is what the exercise teaches.

Of course, you were expected to solve each of these exercises by the tedious and nonsensical method of labeling bar models. As we see, these models are irrelevant for solving these particular exercises, where the “concrete”(as opposed to “pictorial”) is more illuminating.

In the next 2 exercises, the bar models may come handy - mostly as a bookkeeping device, that allows one to figure out rather quickly how to solve them. But the tedious multi-step plan of reading, re-reading, labeling, adjusting simple pictures, would only make the whole process unbearably and unnecessarily long and frustrating. The use of grid paper (instead of blank one) facilitates solving math problems much more than the books we are reviewing (and costs much less). See for yourself:

"Angela baked cookies. She gave 1/3 of them to her sister, put 5/8 of the rest to the fridge ... "

Actually, disobeying the advice of the authors to read the problem to the end (and to do this many times), a curious problem solver would at this point already ask himself: OK - let a segment be the number of cookies, 1/3 of it is given to the sister, and 5/8, of the remaining 2/3 are in the fridge. Hm, but how much is this of the whole? 5/8 of 2/3 is 5/8 times 2/3 = 10/24 =5/12. Well, but 1/3 (i.e. 4/12) and 5/12 add up to 9/12 = 3/4 of all cookies. But what was the problem about?
"... If she had 1 dozen left, how many cookies did she bake?" Aha, if the dozen is 1/4, so the whole is 4 times greater, i.e. 4 x 12=48"
A less curious problem solver would obey the advice and argue differently: If 3/8 of the rest is 12 (a dozen of) cookies, then 1/8 is 4 cookies, and 8/8 is 8x4=32. But this is what is left after 1/3 of all cookies were removed, i.e. it is 2/3 of all cookies. Thus adding a half of it (for 1/3 is a half of 2/3), i.e. 16 we'll get the total 32+16=48.

Either way, one can use drawings of segments, just to have something in front of your eyes and not to forget what you are talking about. Doing it on grid paper may help if you are smart to choose the scale of 8 cells for "the rest" of what's left after 1/3 was removed. But these drawings aren't really necessary. The same is in the last sample problem:

"Marcos wrote 3 pages of science report on Monday, 2/3 of the rest on Tue, and still needed to write 2 more pages. How long is the report?" Well, if 2 remaining pages are 1/3 of what's left after Monday, then what's left after Monday is 3x2=6 pages, and plus 3 pages written on Monday is 9 pages.

Does this require 8 steps of drawing and 1.5 pages of talking? Nope. (And if your opinion is opposite, then - well, it is time to realize that the truth is not a matter of opinion.)

Actually all these samples are chosen incompetently, because they do not illustrate at all the purpose of the book: to show the advantage of model drawing. Here is an exercise from SM grade 6, where drawing pictures can indeed be very useful:

"John has $28 more than Peter. 1/3 of John's money is equal to 4/5th of Peter's money. Find John's money."

On a grid paper, I would draw a strip of 5 cells for Peter's money, and shade four of them as 4/5th of it. These 4 cells are said to be 1/3 of John's money. Thus John's money is 12 cells. Since 12-5 (John's minus Peter's) is 7 cells, then 7 cells are worth $28. Thus 1 cell is worth 28/7=4 dollars. Thus John's money is 12 x $4 = $48.

Let us now go to the last sample page:
http://www.singaporemath.com/v/vspfiles/assets/images/sp_mdswpsw4.gif
It is a page from the "Solution Key" representing the solution to the 4th of the 5 sample problems - about the cookies. It shows that the solution key will also not help you, and that even this page demonstrates its author's incompetence. Here is why.

If it were the "Answer Key", presenting the answer ($48), it would have been useful: You would get some exercises to work on, and a means to find out if the answer you found is correct.
However the solution itself (as opposed to merely the answer) is useless. Actually I already gave you two solutions; so what if the one you found is different from the one the book gives? Does it mean it is wrong? Nope.
In fact I gave you a few lines of reasoning - they might teach you something. What you'll find in the solution key is a picture of what you get at the end of drawing bar models following the 2nd one of my

solutions, plus 15 short lines of computations. The picture is OK, but because you don't know in what order its elements were appearing on the page, it does not tell you the reasoning: how to arrive at the solution. Decoding the picture becomes actually harder than solving the problem on your own. (You may say that it is harder for *me* because I know how solve the exercise. And you'll be right, but that's the reason why *you* should solve problems yourself: *reading* (even my) solutions does not teach you finding yours; don't worry though - SM books teach this.)

While the picture, though useless, is OK, the 15 lines of computations are not, and demonstrate the authors' mathematical incompetence. They abbreviate the word "unit" as "u", and then let it denote many different things at once. In parts A and B of the solution we find that u=4, in part C and D we have u=16. Actually variables used in algebra are exactly the names of various quantities, usually abbreviated to a single letter. It is the simplest requirement of clarity that different quantities must be denoted by different letters within any consistent discourse, be it a research paper, or a section in a textbook. Disobeying this convention within one exercise (and apparently, in each exercise of this book) crosses all borders.
However outrageous, this is in principle a fixable flaw; not the only one though: The suggestion that a rather straightforward exercise on fractions may require 15 lines of calculations is murderous to the whole idea of mathematics education.

Let me conclude by quoting the preface to teacher's guide to the US edition of SM. The preface was written by Madge Goldman, the president of the foundation that arranged for that edition to come out:

"The central idea of all of mathematics is to discover how it is that knowing some few things will, via reasoning, permit us to know much else without having to commit the new information to memory as separate facts. Mathematics is economy of information, not its unnecessary proliferation. Basic mathematics properly presented conveys this lesson. It is the connections, the reasoned, logical connections, that make mathematics manageable. Understanding the structure of mathematics is the key to success. Everyone can be 'good at mathematics,' and this textbook series, as has been proved in Singapore, shows how."

Everyone "good in mathematics" would endorse this description of the subject.
The books under review (actually all books on "model drawing" – including all those yet to be written) go a very-very-very long way in exactly the opposite direction.