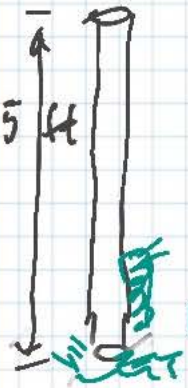
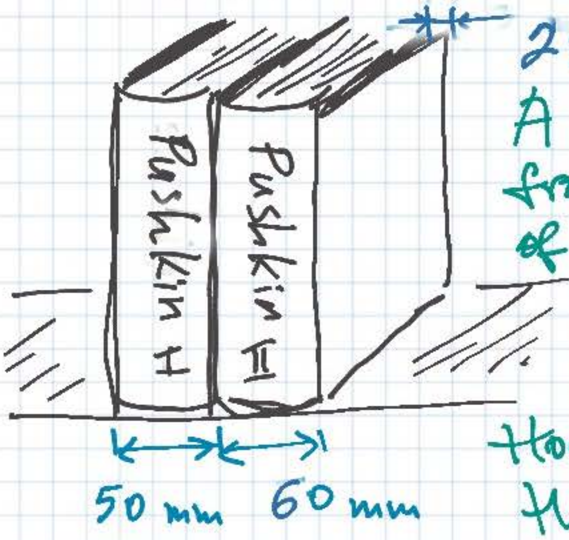


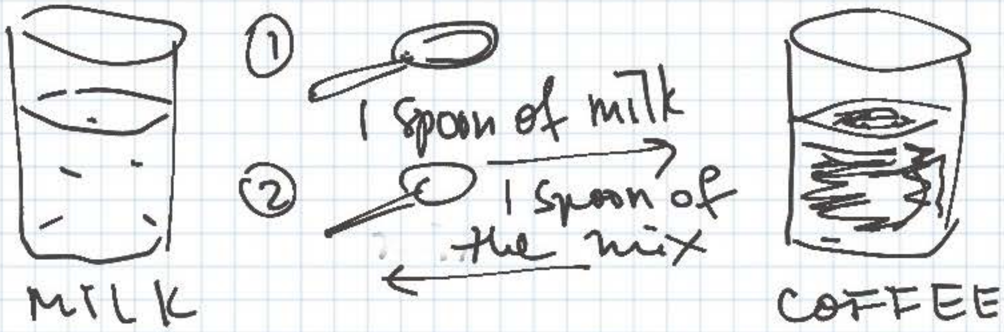
↑ crawls 3 ft up each day
↓ slides 2 ft down each night

In how many days will it reach the top?

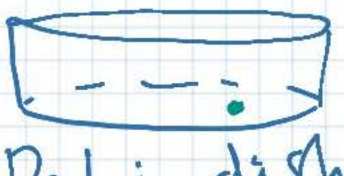


A bookworm ate from the first page of volume I through the last page of volume II.

How long was the hole the bookworm made?



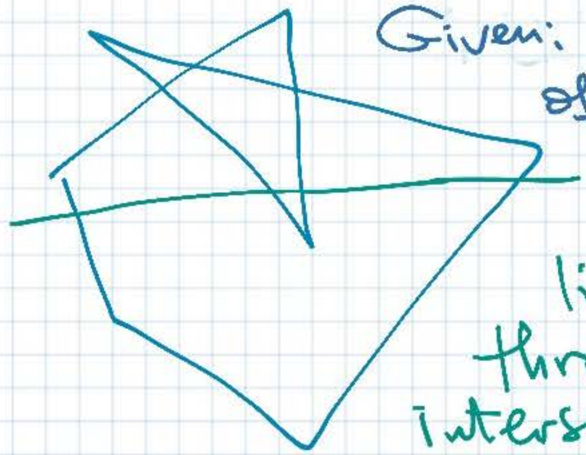
Is there more milk in coffee or coffee in milk?



Bacteria divides every second.

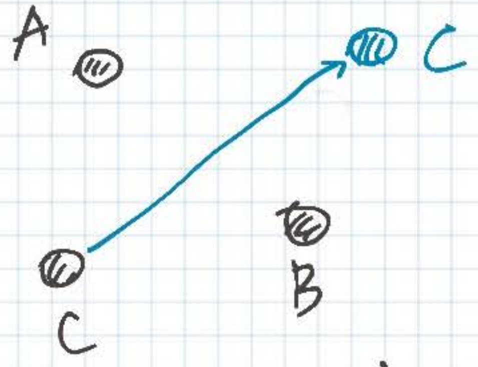
After 1 minute the dish is full.

When was it half-full?



Given: a closed broken line of 7 segments.

Can a straight line not passing through vertices intersect each segment?

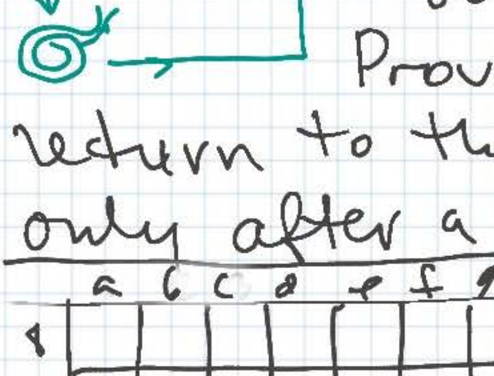


Hockey pucks

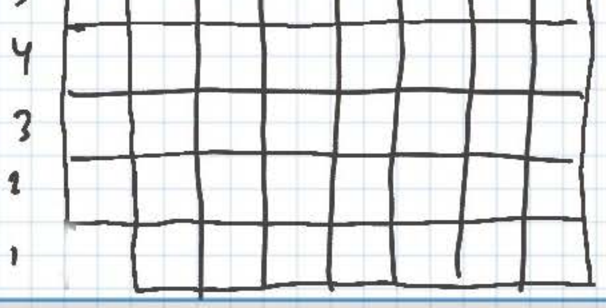
Player hits one so that it passes between the other two.

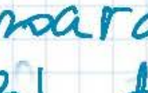
After 25 moves, can all three return to the starting positions?

$1 \pm 2 \pm 3 \pm 4 \pm 5 \pm 6 \pm 7 \pm 8 \pm 9 \pm 10 \overset{?}{=} 0$



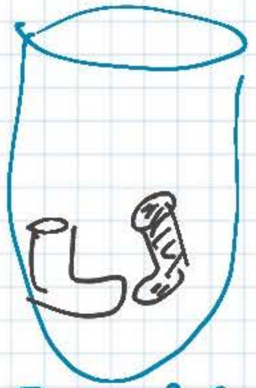
Snail crawls with constant speed, an turns 90° every 15 min. Prove that it can return to the starting point only after a whole number of hours.





domino pieces

Can you tile this chess board with 31 domino pieces?

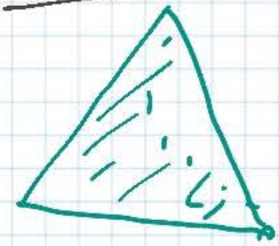


Bag of socks

20 white
20 black

① What is the smallest number of socks that one needs to draw to guarantee a non-pair?

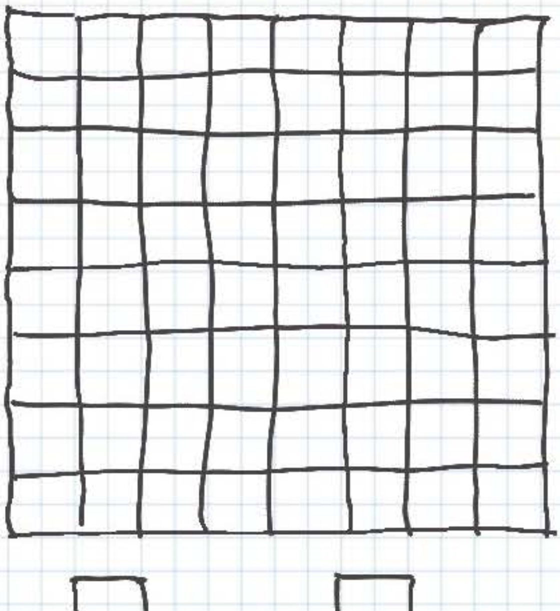
② to guarantee a pair?



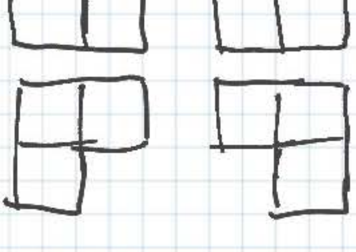
An equilateral triangle cannot be covered by 2 smaller equilateral triangles.

Prove that in any group of 5 people there are two with the same number of friends within the group.

Among 25 apples of 3 sorts, there are at least 9 of the same sort.



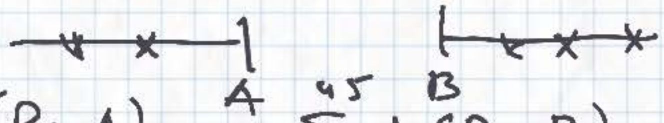
What is the smallest number of squares on the 8×8 board that one needs to color <u>green</u> so that every <u>tromino</u> placed on the board has at least one green square?



trominos

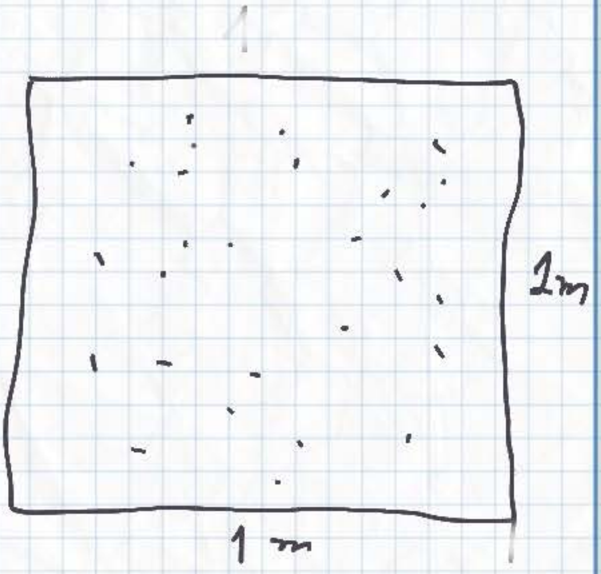
Homework problems:

(1) 45 pts



Prove: $\sum_{i=1}^{45} d(P_i, A) \neq \sum_{i=1}^{45} d(P_i, B)$

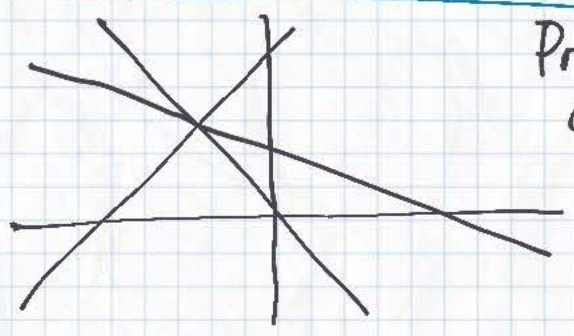
(2) 51 points in



Prove: There is a 20cm × 20 cm square which covers ≥ 3 points

(3) If $2, 5 \nmid n$ then there exists $m = 11\ldots 11$ such that $n \mid m$.

Map of the USF (latland)



Prove: It can be colored in 2 colors so that no two adjacent states have the same color

How many states?

(assuming that no two lines are $\parallel$ nor any three $\divideontimes$)

Induction: $P_1, P_2, \dots, P_n, \dots$ - a sequence of propositions.

Suppose:

1° P_1 is true (Base of induction)

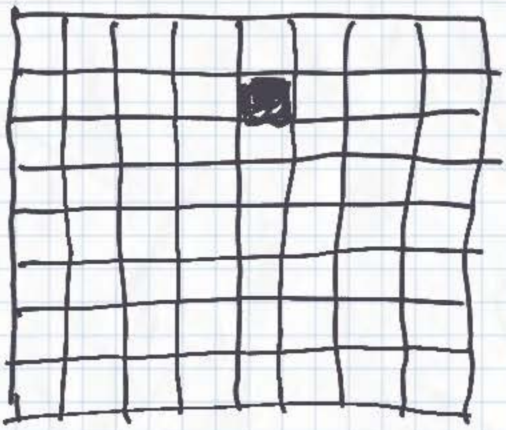
2° For $n \geq 1$, $P_n \Rightarrow P_{n+1}$ is true (Step of induction)

Then P_n is true for all $n \geq 1$.

Proof (?): n - smallest s.t. P_n is false. Then P_{n-1}, $P_{n-1} \Rightarrow P_n$ are true $\Rightarrow$ P_n is true

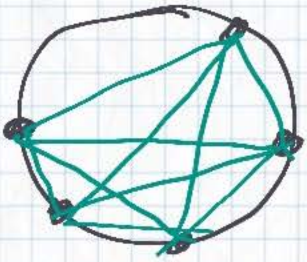
Tromino on the chess board

Tile



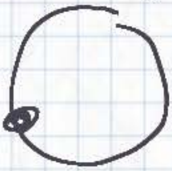
with

Chords in Discoland



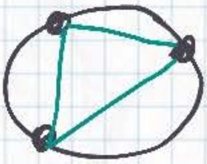
n points are connected by all chords, no 3 ✳.
How many states?

n=1

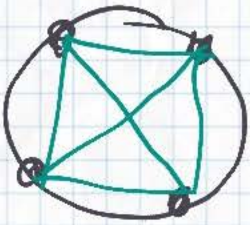


n = 2

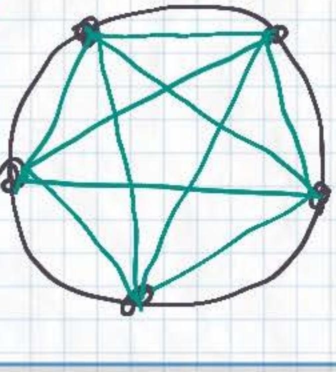
n = 3



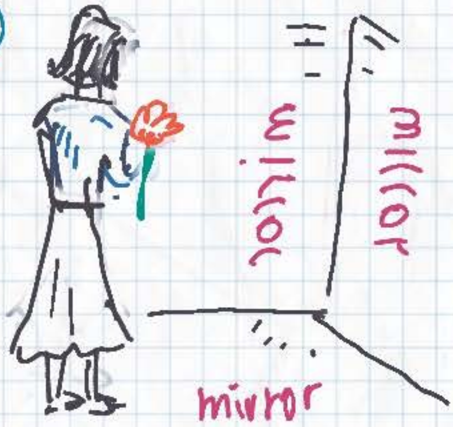
n = 4



n=5

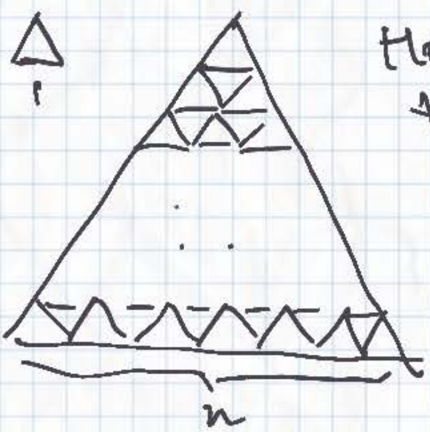


(1)



How many flowers will you see?

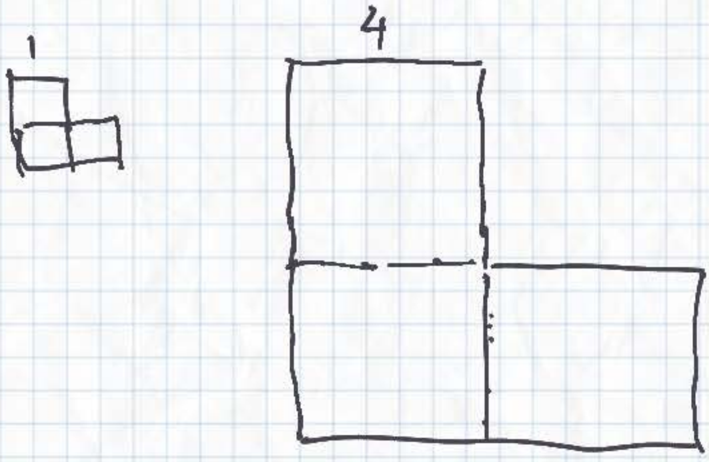
(2)



How many small triangles are in a big one?

(3) $1 + 3 + 5 + \dots 2n - 1 = ?$

(4)



How many small trominos is needed to tile the big one?

3.2

(5) $1+4+9+\dots+n^2 = \,? =: f(n)$

$$Df(n) := f(n) - f(n-1) = n^2$$

$$\to V_k \xrightarrow{D} V_{k-1} \to \dots$$

Space of polynomials of degree $\leq k$

$\underline{\dim}\ V_k = \,?$

$\underline{\text{nullity}}$ of $D = \,?$

$\underline{\text{rank}}$ of $D = \,?$

$$Dn^k = kn^{k-1} + \left(\begin{matrix}\text{terms of}\\ \text{lower degree}\end{matrix}\right)$$

$$\begin{array}{r} n^k \\ -\ (n-1)^k = n^k - kn^{k-1} + \dots \\ \hline kn^{k-1} + \dots \end{array}$$

$$Df = n^2 \implies$$

$$f = \frac{n^3}{3} + An^2 + Bn + C$$

n	$f(n)$
0	0
1	1
2	5

$\Rightarrow C = 0$

$\Rightarrow A + B = \frac{2}{3}$ $\left(\Rightarrow B = \frac{1}{6}\right)$

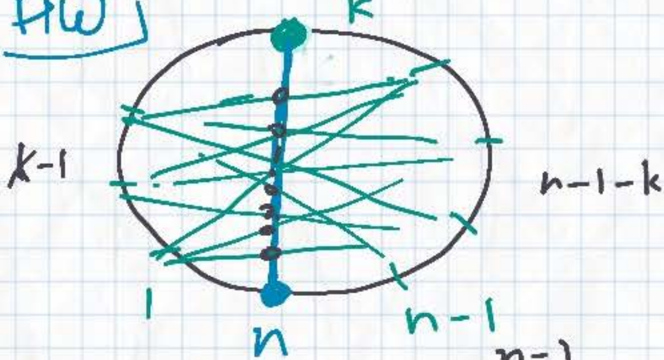
$\Rightarrow \frac{8}{3} + \frac{4}{3} + 2A = 5 \Rightarrow A = \frac{1}{2}$

$$f(n) = \frac{2n^3 + 3n^2 + n}{6} = \frac{n(n+1)(2n+1)}{6}$$

$$f(3) = \frac{3 \cdot 4 \cdot 7}{6} = 14$$

HW

3.3



$$f(n) - f(n-1) = \sum_{k=1}^{n-1} (k-1)\underbrace{(n-1-k)}_{}^{(n-2)-(k-1)}$$

$$= (n-2)\sum_{k=1}^{n-1}(k-1) - \sum_{k=1}^{n-1}(k-1)^2$$

$$= \frac{n^3}{2} - \frac{n^3}{3} + (\text{lower order terms})$$

$$f(n) = \frac{n^4 + An^3 + Bn^2 + Cn + D}{24}$$

n	$f(n)$
0	1
1	1
2	2
3	4

$$\Rightarrow f(n) = 1 + \frac{n(n-1)}{24}\left(n^2 + \alpha n + \beta\right)$$

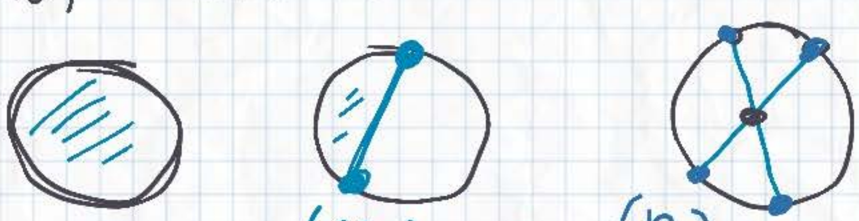
$$\Rightarrow \left.\begin{array}{l}(4 + 2\alpha + \beta) = 12 \\ (9 + 3\alpha + \beta) = 12\end{array}\right\} \Rightarrow \begin{array}{l}\alpha = -5 \\ \beta = 18\end{array}$$

$$f(n) = 1 + \frac{n(n-1)}{2} + \frac{n(n-1)(n-2)(n-3)}{24}$$

$$f(4) = 1 + 6 + 1 = 8$$

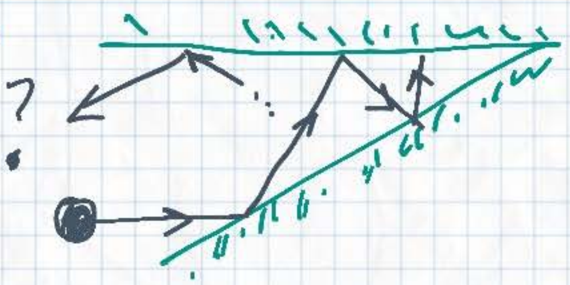
$$f(5) = 1 + 10 + 5 = 16$$

$$f(6) = 1 + 15 + 15 = 31$$

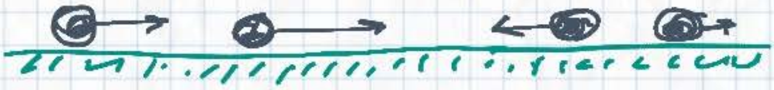


$$1 + \binom{n}{2} + \binom{n}{4}$$

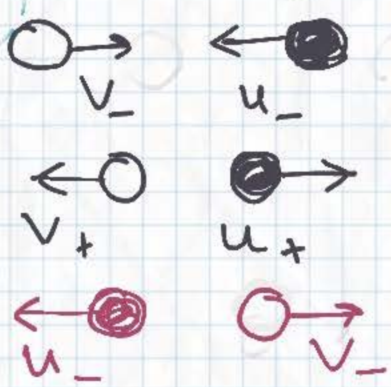
(1)



(2)



How many collisions?



$$m v_- + m u_- = m v_+ + m u_+$$

$$\frac{m v_-^2}{2} + \frac{m u_-^2}{2} = \frac{m v_+^2}{2} + \frac{m u_+^2}{2}$$

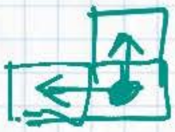
$$u_+ = v_- \qquad v_+ = u_-$$

(3) $D^k f = \delta$ (Is a solution unique?)

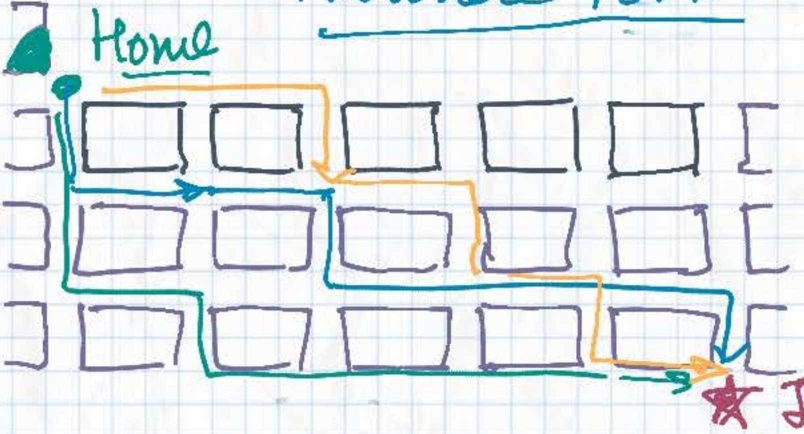
n	-1	0	1	2	3	
$\delta(n)$	0	1	0	0	0	
...	0	1	1	1	1	...
...	0	1	2	3	4	...
...	0	1	3	6	10	...
...	0	1	4	10	20	...
...	0	1	5	15	35	...

$$f_k(n) = \binom{n-1+k}{k} = \binom{n+k-1}{k-1}$$

$$f_k(n) = f_k(n-1) + f_{k-1}(n)$$



Nowhere York



How many shortest routes ● ⟶ ☆ are there?

Pascal's Triangle

row sum 1 | 1 | row ± sum 1
2 | 1 1 | 0
4 | 1 2 1 | 0
8 | 1 3 3 1 | 0
… | 1 4 6 4 1 | ⋮

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

. . . .

$$(1+x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k = (1+x)(1+x)\dots(1+x)$$

$$(1+x)^n = (1+x)(1+x)^{n-1}$$

$$\binom{n}{k} = \binom{n-k}{k} + \binom{n-1}{k-1}$$

coeff at x^k

n	0	1	2	3	4	
$\sum_{k=0}^{n}\binom{n}{k}^2$	1	2	6	20	70	… ?