Math 54-1 Your name: Key
Quiz 9, July 27, 2010

Please write your name **on each sheet**. Show your work clearly and in order, including intermediate steps in the solutions and the final answer.

1. (6 pt) The matrix

$$A = \begin{bmatrix} 17 & -12 \\ 24 & -17 \end{bmatrix}$$

has characteristic polynomial $\lambda^2 - 1$. Compute the power A^{137}. Explain your steps carefully. Your answer should be simplified to contain only the decimal digits 0–9, the minus sign, and the square brackets.

A has 2 eigenvalues 1 and -1, both w/ multiplicity 1.
Since A is 2×2, it is diagonalizable:

$$A = P\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}P^{-1} \text{ for some invertible } P.$$

Then,

$$A^{137} = P\begin{bmatrix} 1^{137} & 0 \\ 0 & (-1)^{137} \end{bmatrix}P^{-1} = P\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}P^{-1} = A.$$

So,

$$\boxed{A^{137} = \begin{bmatrix} 17 & -12 \\ 24 & -17 \end{bmatrix}}.$$

(You did not need to compute that, but

$$P = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix},$$

$$P^{-1} = \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}.)$$

2. (4 pt) Suppose that $\mathcal{B} = \{\vec{b}_1, \vec{b}_2\}$ and $\mathcal{C} = \{\vec{c}_1, \vec{c}_2\}$ are two bases of a vector space V, and

$$\vec{b}_1 = 6\vec{c}_1 - 2\vec{c}_2, \ \vec{b}_2 = 9\vec{c}_1 - 4\vec{c}_2.$$

A vector $\vec{x} \in V$ has $[\vec{x}]_{\mathcal{B}} = (1, 2)$. Find $[\vec{x}]_{\mathcal{C}}$.

Solution 1: $P_{\mathcal{C}\leftarrow\mathcal{B}} = [[\vec{b}_1]_{\mathcal{C}} \ [\vec{b}_2]_{\mathcal{C}}] = \begin{bmatrix} 6 & 9 \\ -2 & -4 \end{bmatrix}$;

$[\vec{x}]_{\mathcal{C}} = P_{\mathcal{C}\leftarrow\mathcal{B}} [\vec{x}]_{\mathcal{B}} = \begin{bmatrix} 6 & 9 \\ -2 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 24 \\ -10 \end{bmatrix}$

Solution 2: $[\vec{x}]_{\mathcal{B}} = 6\begin{bmatrix} 1 \\ 2 \end{bmatrix} \rightarrow \vec{X} = \vec{b}_1 + 2\vec{b}_2 =$

$= (6\vec{c}_1 - 2\vec{c}_2) + 2(9\vec{c}_1 - 4\vec{c}_2) = 24\vec{c}_1 - 10\vec{c}_2 \Rightarrow$

$\Rightarrow [\vec{x}]_{\mathcal{C}} = \begin{bmatrix} 24 \\ -10 \end{bmatrix}$.