

Math 54-1

Quiz 1, June 29, 2010

Your name: Key

Please write your name on each sheet. Show your work clearly and in order, including intermediate steps in the solutions and the final answer.

1. (5 pt) Solve the following system of linear equations:

$$x_1 + 2x_2 + x_3 = 5,$$

$$2x_1 + 4x_2 = 4.$$

State all row operations clearly and display intermediate steps.

Augmented matrix: $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 5 \\ 2 & 4 & 0 & 4 \end{array} \right] \xrightarrow{R_2 \leftarrow R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 5 \\ 0 & 0 & -2 & -6 \end{array} \right]$

$$\xrightarrow{R_2 \leftarrow -\frac{R_2}{2}} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 5 \\ 0 & 0 & 1 & 3 \end{array} \right] \xrightarrow{R_1 \leftarrow R_1 - R_2} \left[\begin{array}{ccc|c} 1 & 2 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

$$\left. \begin{array}{l} x_1 + 2x_2 = 2 \\ x_3 = 3 \end{array} \right\}$$

$$\begin{array}{l} x_1 = 2 - 2x_2 \\ x_3 = 3 \\ x_2 \text{ free} \end{array}$$

2. (5 pt) Does \vec{b} lie in $\text{Span}\{\vec{a}_1, \vec{a}_2\}$, where

$$\vec{a}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \vec{a}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \vec{b} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}?$$

$\vec{b} \in \text{Span}\{\vec{a}_1, \vec{a}_2\} \Leftrightarrow$ the system with the augmented matrix $[\vec{a}_1 \ \vec{a}_2 \ \vec{b}]$ ~~has~~ is consistent.

Row reduce: $[\vec{a}_1 \ \vec{a}_2 \ \vec{b}] = \begin{bmatrix} 0 & 0 & 3 \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ $\begin{matrix} R_1 \leftarrow R_2 \\ R_2 \leftarrow R_3 \\ R_3 \leftarrow R_1 \end{matrix}$

$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 3 & 2 \end{array} \right]$ Inconsistent, as there is a pivot in last column.

So, $b \notin \text{Span}\{\vec{a}_1, \vec{a}_2\}$.