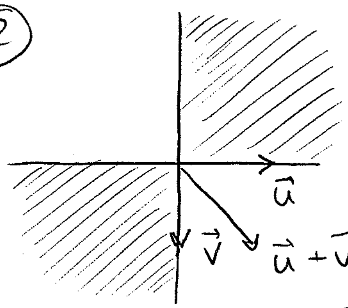
MATH 54-1, H/W 6

4.1 ② 

ⓐ Yes: $u \in W \iff u_1 \cdot u_2 \geq 0$, but $cu = \begin{bmatrix} cu_1 \\ cu_2 \end{bmatrix}$, $cu_1 \cdot cu_2 = c^2 u_1 u_2 \geq 0 \to cu \in W$

ⓑ $\vec{u} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

⑥ Not a subspace, as it does not contain the zero polynomial: $a+t^2$ is not the zero polynomial for any a.

⑫ We have $W = \text{Span}\{\vec{u}, \vec{v}\}$, where $\vec{u} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 3 \\ -1 \\ -1 \\ 4 \end{bmatrix}$.

By Theorem 1, W is a subspace of $\mathbb{R}^4$.

(24) ⓐ True, by definition (p. 217) ⓑ True, by (3) on p. 217

ⓒ True; for example, a vector space is a subspace of itself

ⓓ False, see Example 8 ⓔ False. For example, the second condition should read, "for all $u, v \in H$, we have $u+v \in H$".

(32) We know:

$(1)_H \quad 0 \in H$ $\qquad (1)_K \quad 0 \in K$

$(2)_H \quad u, v \in H \Rightarrow u+v \in H$ $\qquad (2)_K \quad u, v \in K \Rightarrow u+v \in K$

$(3)_H \quad u \in H, c \in \mathbb{R} \Rightarrow cu \in H$ $\qquad (3)_K \quad u \in K, c \in \mathbb{R} \Rightarrow cu \in K$

We need to prove:

$(1)_{H\cap K}$ $0 \in H \cap K$. Indeed, $0 \in H$ and $0 \in K$.

$(2)_{H\cap K}$ $u, v \in H\cap K \Rightarrow u+v \in H\cap K$. Indeed,

$$\left.\begin{array}{l} u,v \in H \to \text{by } (2)_H, \ u+v \in H \\ u,v \in K \to \text{by } (2)_K, \ u+v \in K \end{array}\right\} \text{So, } u+v \in H\cap K$$

$(3)_{H\cap K}$ $u \in H\cap K$, $c \in \mathbb{R} \to cu \in H\cap K$. Indeed,

$$\left.\begin{array}{l} u \in H \to \text{by } (3)_H, \ cu \in H \\ u \in K \Rightarrow \text{by } (3)_K, \ cu \in K \end{array}\right\} \text{so, } cu \in H\cap K.$$

4.2 ⑧ Not a ~~vector~~ sub space of $\mathbb{R}^3$, as does not contain the zero vector. Indeed, if $r = s = t = 0$, then $5r-1 \neq s+2t$.

⑫ Not a subspace of $\mathbb{R}^4$, as does not contain the zero vector. Indeed, otherwise there ~~is~~ are b, d: $b-5d = 2b = 2d+1 = d = 0$. But $d = 0 \to 2d+1 \neq 0$, a contradiction.

4.3 (14) Basis for Col A → pivot Columns of A; so,

$\begin{bmatrix}1\\2\\1\\3\end{bmatrix}, \begin{bmatrix}-5\\-5\\0\\-5\end{bmatrix}, \begin{bmatrix}-3\\2\\5\\-2\end{bmatrix}.$

Basis for Nul A: find the parametric vector description of the solution set of $A\vec{x}=0$.

Row reduce $A \to B \to \begin{bmatrix}1 & 2 & 0 & 4 & 0\\0 & 0 & 1 & -7/5 & 0\\0 & 0 & 0 & 0 & 1\\0 & 0 & 0 & 0 & 0\end{bmatrix} \to$

→ Basis for Nul A is $\begin{bmatrix}-2\\1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}-4\\0\\7/5\\1\\0\end{bmatrix}$

(30) Let $A=[\vec{v}_1, \dots, \vec{v}_k]$

Then A has n(#rows) < k (#columns). So, there cannot be a pivot position in every column ~~row~~ of A. So, $\vec{v}_1, \dots, \vec{v}_n$ cannot be linearly independent and thus ~~cannot~~ do not form a basis.