Math 54, Section 214 Your name: Key

Quiz 3, February 12, 2010

Please write your name **on each sheet**. Show your work clearly and in order, including intermediate steps in the solutions and the final answer.

1. (7 pt) Compute $\mathrm{T}^{-1}\mathrm{AT}$, if

$$\mathrm{A} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \ \mathrm{T} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}.$$

We have $T^{-1} = \dfrac{1}{1\cdot 1 + 1\cdot 1}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = \dfrac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix};$

$$T^{-1}AT = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

2. (7 pt) Is the following matrix invertible? Explain. Do NOT use determinants.

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Row reduction: $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \xrightarrow{R_2 \leftarrow R_2 - R_1} \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \rightarrow$

$\xrightarrow{R_3 \leftarrow R_3 + R_2} \begin{bmatrix} \boxed{1} & 1 & 0 \\ 0 & \boxed{-1} & 1 \\ 0 & 0 & \boxed{2} \end{bmatrix}$. Since there are 3 pivot positions, the matrix is invertible.

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3. (6 pt) Given the matrix A and its reduced row echelon form, find a basis for $\operatorname{Col} A$ and a basis for $\operatorname{Nul} A$:

$$A = \begin{bmatrix} 0 & 2 & 1 & 7 \\ 0 & 2 & 2 & 10 \\ 0 & 1 & 1 & 5 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The basis for Col A is given by the pivot columns of A:

$$\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

To find a basis for Nul A, we solve the equation $A\vec{x}=0$:

$$\begin{cases} x_2 + 2x_4 = 0 \\ x_3 + 3x_4 = 0 \\ x_1, x_4 \text{ free} \end{cases} \Rightarrow \vec{x} = \begin{bmatrix} x_1 \\ -2x_4 \\ -3x_4 \\ x_4 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ -2 \\ -3 \\ 1 \end{bmatrix};$$

A basis for Nul A is $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ -3 \\ 1 \end{bmatrix}$