Math 1B quiz 1

Sep 2, 2009

Please write your solutions on this sheet, continuing on a separate sheet if necessary. Write your name in the upper right corner of **every** sheet you submit. Please include the intermediate steps in your solutions.

Section 105

Compute the following integrals.

1. (3 pt) $\int x \cos 5x \, dx$
2. (3 pt) $\int_0^3 \frac{x}{\sqrt{x+1}} \, dx$
3. (4 pt) $\int x^5 e^{-x^2} \, dx$

Section 106

Compute the following integrals.

1. (3 pt) $\int \cos x \ln(\sin x) \, dx$
2. (3 pt) $\int_1^2 x\sqrt{x-1} \, dx$
3. (4 pt) $\int x^5 \cos(4x^2) \, dx$

Solutions for section 105

1. First, make the linear substitution $t = 5x$; then $x = \frac{t}{5}$, $dx = \frac{1}{5}dt$, and

$$\int x \cos 5x \, dx = \frac{1}{25}\int t \cos t \, dt.$$

Now, integrate by parts using $u = t$, $dv = \cos t \, dt$; then $du = dt$, $v = \sin t$, and

$$\int t \cos t \, dt = t \sin t - \int \sin t \, dt = t \sin t + \cos t + C.$$

Substituting $t = 5x$, we get

$$\int x \cos 5x \, dx = \frac{1}{5} x \sin 5x + \frac{1}{25} \cos 5x + C.$$

(Alternatively, first integrate by parts and then use the linear substitution.)

2. Make the linear substitution $u = x + 1$; then $x = u - 1$, $dx = du$, and

$$\int_0^3 \frac{x}{\sqrt{x+1}} \, dx = \int_1^4 \frac{u-1}{\sqrt{u}} \, du = \int_1^4 u^{1/2} - u^{-1/2} \, du.$$

Now,

$$\int u^{1/2} - u^{-1/2} \, du = \frac{2}{3}u^{3/2} - 2u^{1/2} + C,$$

so by the Fundamental Theorem of Calculus,

$$\int_1^4 u^{1/2} - u^{-1/2} \, du = \frac{2}{3}(4^{3/2} - 1^{3/2}) - 2(4^{1/2} - 1^{1/2}) = \frac{8}{3}.$$

3. First, make the substitution $t = -x^2$; then $dt = -2x \, dx$ and

$$\int x^5 e^{-x^2} \, dx = -\frac{1}{2}\int t^2 e^t \, dt.$$

Now, integrate by parts twice with $dv = e^t \, dt$, $v = e^t$:

$$\begin{aligned} \int t^2 e^t \, dt &= t^2 e^t - 2\int t e^t \, dt \\ &= t^2 e^t - 2te^t + 2\int e^t \, dt \\ &= t^2 e^t - 2te^t + 2e^t + C \\ &= e^t(t^2 - 2t + 2) + C. \end{aligned}$$

Substituting $t = -x^2$, we get

$$\int x^5 e^{-x^2} \, dx = -\frac{e^{-x^2}}{2}(x^4 + 2x^2 + 2) + C.$$

Solutions for section 106

1. Make the substitution $t = \sin x$; then $dt = \cos x\, dx$ and

$$\int \cos x \ln(\sin x)\, dx = \int \ln t\, dt.$$

Then integrate by parts with $u = \ln t$, $dv = dt$: we have $du = \frac{1}{t}\, dt$, $v = t$, and

$$\int \ln t\, dt = t \ln t - \int dt = t \ln t - t + C.$$

Substituting $t = \sin x$, we get

$$\int \cos x \ln(\sin x)\, dx = \sin x(\ln(\sin x) - 1) + C.$$

(Alternatively, first integrate by parts and then use the substitution.)

2. Make the linear substitution $u = x - 1$; then $x = u + 1$, $dx = du$, and

$$\int_1^2 x\sqrt{x-1}\, dx = \int_0^1 (u+1)\sqrt{u}\, du = \int_0^1 u^{3/2} + u^{1/2}\, du.$$

Now, $\int u^{3/2} + u^{1/2}\, du = \frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2} + C$, so

$$\int_0^1 u^{3/2} + u^{1/2}\, du = \frac{2}{5}(1^{5/2} - 0^{5/2}) + \frac{2}{3}(1^{3/2} - 0^{3/2}) = \frac{16}{15}.$$

3. First, make the substitution $t = 4x^2$; then $x^2 = t/4$, $dt = 8x\, dx$ and

$$\int x^5 \cos(4x^2)\, dx = \frac{1}{128}\int t^2 \cos t\, dt.$$

Now, we integrate by parts twice. First we put $u = t^2$, $dv = \cos t\, dt$; then $du = 2t\, dt$, $v = \sin t$, and

$$\int t^2 \cos t\, dt = t^2 \sin t - 2\int t \sin t\, dt.$$

Now, put $u = t$, $dv = \sin t\, dt$; then $du = dt$, $v = -\cos t$, and

$$\begin{aligned} \int t^2 \cos t\, dt &= t^2 \sin t + 2t\cos t - 2\int \cos t\, dt \\ &= t^2 \sin t + 2t \cos t - 2 \sin t + C. \end{aligned}$$

Substituting $t = 4x^2$, we get

$$\int x^5 \cos(4x^2)\, dx = \frac{1}{8}x^4 \sin(4x^2) + \frac{1}{16}x^2\cos(4x^2) - \frac{1}{64}\sin(4x^2) + C.$$