Math 1B, Section 105 Your name: Key

Quiz 7, October 14, 2009

Please write your name on each sheet. Show your work clearly and in order, including the intermediate steps in the solutions and the final answer.

1. (5 pt) Consider the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{\sqrt{n}}.$$

(a) Find the radius of convergence and the interval of convergence.

(b) Differentiate the series to obtain a new series.

(c) Find the interval of convergence of the differentiated series. (You may use that this series has the same radius of convergence as the original series.)

(a)

Ratio Test: $\lim_{n\to\infty} \frac{\left|\frac{(-1)^{n+1} x^{n+1}}{\sqrt{n+1}}\right|}{\left|\frac{(-1)^n x^n}{\sqrt{n}}\right|} = \lim_{n\to\infty} \frac{|x|}{\sqrt{1+\frac{1}{n}}} = |x|$

$|x|<1 \to$ series converges

$|x|>1 \to$ series diverges

$\boxed{R=1}$

Endpoints: $x=1 \to \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

Converges by Alt. Series Test: $0 \le \frac{1}{\sqrt{n+1}} \le \frac{1}{\sqrt{n}}$, $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$.

$x=-1 \to \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

Diverges by p-series test

Interval of convergence: $(-1, 1]$

(b) Differentiate:

$$\sum_{n=1}^{\infty} \frac{(-1)^n n x^{n-1}}{\sqrt{n}} = \sum_{n=1}^{\infty} \sqrt{n}\cdot(-1)^n x^{n-1}$$

$\boxed{R=1}$, same as for the original series

Endpoints: $x=1 \to \sum_{n=1}^{\infty} (-1)^n \sqrt{n}$

$x=-1 \to -\sum_{n=1}^{\infty} \sqrt{n}$

Both diverge by Test for Divergence

$\Downarrow$

$\boxed{\text{Interval of convergence: } (-1, 1)}$

2. (5 pt) Find a power series representation (in terms of powers of x) of the function

$$f(x) = \ln(1 - x^2).$$

Determine the radius of convergence of the series.

$$f'(x) = -\frac{2x}{1-x^2} = -2x \cdot \frac{1}{1-x^2} =$$

$$= -2x \cdot \sum_{n=0}^{\infty} (x^2)^n = -2x \cdot \sum_{n=0}^{\infty} x^{2n},$$

Converges for $|x^2| < 1 \iff |x| < 1$ $\boxed{R=1}$

$$f(x) = \cancel{C+} \int f'(x)\,dx = -2\int \sum_{n=0}^{\infty} x^{2n+1}\,dx =$$

$$= -2\sum_{n=0}^{\infty} \frac{x^{2n+2}}{2n+2} = -\sum_{n=0}^{\infty} \frac{x^{2n+2}}{n+1} =$$

$$= -x^2 - \frac{x^4}{2} - \frac{x^6}{3} - \dots$$

Same R of convergence

$$\boxed{f(x) = -\sum_{n=0}^{\infty} \frac{x^{2n+2}}{n+1} \quad \boxed{R=1}}$$