

Math 1B practice final exam *

Dec 13, 2009

1. (10 pt) Determine whether the improper integral $\int_1^{\infty} \frac{d}{dx}(e^{-x} \ln x) dx$ converges, and if it does, find its value.
2. (10 pt) Perform the integration: $\int \frac{x^2+2x-1}{x(x^2+1)} dx$.
3. (10 pt) Perform the integration: $\int \cos^3 x \sin^3 x dx$.
4. (15 pt) Find the area of the surface of revolution one obtains by revolving about the x -axis the portion of the curve $y^2 = e^x + 1$ that lies between the lines $x = 0$ and $x = 1$.
5. (15 pt) Do the following infinite series converge?
 - (a) $\sum_{n=1}^{\infty} (n^{-2} + 2^{-n})$;
 - (b) $\sum_{n=1}^{\infty} e^{-\sqrt{n}}$.
6. (15 pt) Let k be a real number.
 - (a) Find the n -th Taylor coefficient of the function $f(x) = (1+x)^k$ about the point $x = -1/2$.
 - (b) Determine the radius of convergence of the Taylor series of f about the point $x = -1/2$.
7. (10 pt) Find the general solution of the differential equation $(x^2 + 4x + 5)^{1/2} y' = y^2$.
8. (15 pt) Let a vibrating system consist of a weight attached to a spring. Assume that, in suitable units, the mass of the weight is 1, the damping constant is 2, and the spring constant is 2. Assume that the system is subjected to the driving force $F(t) = F_0 \sin t$ (F_0 is a constant), and that it starts from the rest in its equilibrium position at time $t = 0$. Find the formula giving the displacement as a function of time.
9. What is the 46th derivative of the function $f(x) = e^{-x^2}$ at the origin?
10. Find a numerical approximation to $\int_0^{1/10} \cos(x^2) dx$ accurate to within 10^{-10} . Explain your method.

*The problems are taken from Math 1B final exam in Spring 1996 semester by Prof. Donald Sarason.

11. (a) Use the method of power series to solve the initial value problem

$$(1 - x^2)y'' - 6xy' - 4y = 0, \quad y(0) = 1, \quad y'(0) = 0.$$

(b) What is the radius of convergence of the series you obtain?

(c) Can you sum the series?