Twizzes (theory quizzes)

Math 1B, sections 105/106

Once in a while in the beginning of class we will have short (2–5 minutes depending on the content) theoretical quizzes on the material studied in lecture the previous week(s). You will be given index cards and asked to write several theoretical facts (definitions, statements of mathematical facts, etc.) and then hand them back. **Twizzes (unlike the quizzes that we will have each Wednesday at the end of class) will not be graded; in fact, I ask you not to write your names on the index cards.**

The purpose of these twizzes is to make sure that you know some facts that will be used in discussion sections and that you can state them (a skill which will prove useful in many other courses). I will provide reference to the parts of the textbook with the required material and ask you to read this material before class. This document will include sample answers to all questions; it is OK to learn them by heart and copy to the index cards, or you may choose to make your own statements. However, your answers should be clear enough for a person who forgot the rule to remember it and be able to use it to solve problems. **Strictly speaking, the provided sample answers are incomplete because they lack certain hypotheses (e.g., continuity or differentiability of the functions involved); they are given as minimal pieces of information enough to remember the rules.**

Twiz 1: Wed, Sep 2

1. The substitution rule: Stewart, section 5.5, the red box on page 401.
   If $f(x)$ is a function and we have a substitution $u = g(x)$, then

$$\int f(g(x))g'(x)\,dx = \int f(u)\,du.$$

2. Integration by parts: Stewart, section 7.1, the material before Example 1.
   If $f(x)$ and $g(x)$ are two functions, then

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int g(x)f'(x)\,dx.$$

Twiz 2: Mon, Sep 14

In all integration rules below,

- $\int_a^b f(x)\,dx$ is the integral we compute,
- n is the number of subintervals we divide $[a, b]$ into,
- $\Delta x = (b - a)/n$ is the length of each subinterval, and
- $x_i = a + i \cdot \Delta x$ is the right endpoint of ith subinterval and the left endpoint of $i + 1$st subinterval. (Here i is an integer number between 0 and n.)

1. Midpoint Rule: Stewart, section 7.7, the red box on page 496.

$$\int_a^b f(x)\,dx \approx \Delta x \left(f\left(\frac{x_0 + x_1}{2}\right) + f\left(\frac{x_1 + x_2}{2}\right) + \cdots + f\left(\frac{x_{n-1} + x_n}{2}\right)\right).$$

2. Trapezoidal Rule: Stewart, section 7.7, the red box on page 497.

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}(f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)).$$

3. Simpson's Rule: Stewart, section 7.7, the red box on page 502.

$$\begin{aligned}\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}(&f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots \\ &+2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)).\end{aligned}$$

Twiz 3: Mon, Sep 21

1. Arc length formula: Stewart, section 8.1, red box on page 526.

If f is a function, then the length of the curve $y = f(x)$, $a \leqslant x \leqslant b$, is given by the formula

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx.$$

2. Area of a surface of revolution: Stewart, section 8.2, first red box on page 534.

If f is a positive function, then the area of the surface obtained by rotating the curve $y = f(x)$, $a \leqslant x \leqslant b$, around the x-axis is

$$S = 2\pi \int_a^b f(x)\sqrt{1 + [f'(x)]^2}\,dx.$$

Twiz 4: Mon, Sep 28

1. Integral comparison test: Stewart, section 11.3, red box on page 699.

If f is a continuous positive decreasing function on $[1, \infty)$ and $a_n = f(n)$ for any positive integer n, then the series $\sum_{n=1}^{\infty} a_n$ converges if and only if the integral $\int_1^{\infty} f(x)\,dx$ converges.

2. Test for divergence: Stewart, section 11.2, second red box on page 692.

If $\lim_{n\to\infty} a_n$ does not exist or is nonzero, then $\sum_{n=1}^{\infty} a_n$ diverges.

Twiz 5: Mon, Oct 5

1. Comparison test for series: Stewart, section 11.4, red box on page 705.

Suppose that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are two series with nonnegative terms, and $a_n \leqslant b_n$ for all n. Then: (1) if $\sum b_n$ is convergent, so is $\sum a_n$; (2) if $\sum a_n$ is divergent, then so is $\sum b_n$.

2. Limit comparison test for series: Stewart, section 11.4, red box on page 707.

Suppose that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are two series with nonnegative terms, and $\lim_{n\to\infty} \frac{a_n}{b_n}$ exists and lies strictly between 0 and ∞. Then $\sum a_n$ converges if and only if $\sum b_n$ converges.

Twiz 6: Mon, Oct 12

1. Ratio test: Stewart, section 11.6, red box on page 716.

Let $L = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$. Then:

- if $L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ converges absolutely;
- if $L > 1$, then the series diverges;
- if $L = 1$, then the test is inconclusive.

2. Root test: Stewart, section 11.6, red box on page 718.

Let $L = \lim_{n\to\infty} \sqrt[n]{|a_n|}$. Then:

- if $L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ converges absolutely;
- if $L > 1$, then the series diverges;
- if $L = 1$, then the test is inconclusive.