QUALIFYING EXAMINATION FOR KIRIL DATCHEV

M. CHRIST, L.C. EVANS, H. MURAYAMA (PHYSICS), M. ZWORSKI

Major Topic: Harmonic Analysis (classical analysis)

Distribution Theory. Test functions. Approximate identities. Borel's lemma. Distributions: order, convergence, support, differentiation, convolution. Homogeneous distributions. (from Hörmander, Chapters 1-4)

Fourier Transform. Schwartz space. Tempered distributions. Convergence. Fourier transform on $\mathcal{S}$, on $\mathcal{S}'$, on L^1, on L^2. Fourier inversion. Poisson summation formula and periodic distributions, consequences for Fourier series. (from Hörmander, Chapter 7)

Interpolation. Marcinkiewicz interpolation. Riesz-Thorin interpolation. Application to Young's inequality, Hausdorff-Young theorem. (from Christ, Chapter 3)

Hardy-Littlewood Maximal Function. Vitali covering lemma, L^p boundeddness. Application to Lebesgue differentiation theorem and to solving Laplace's equation on a half space. (from Christ, Chapter 3)

Singular Integral Operators and Littlewood-Paley Theory. Calderón-Zygmund decomposition. L^p boundedness of Calderón-Zygmund operators. L^p boundedness of homogeneous multipliers. Almost everywhere existence of principal value integrals. Almost everywhere differentiability. Cotlar-Knapp-Stein lemma. Almost orthogonality. Littlewood-Paley estimates. (from Christ, Chapters 4-5)

Oscillatory Integrals. Method of stationary phase. Airy functions. Van der Corput estimates. Boundedness of oscillatory integral operators. (from Christ, Chapter 6)

Major Topic: Semiclassical Analysis (classical analysis)

Local Symplectic Geometry. Lie derivative of a differential form, Cartan's formula, Poincaré's lemma. Symplectic forms, symplectomorphisms, generating functions. Hamiltonian vector fields, Poisson bracket. Jacobi's theorem. Darboux's theorem. Deformation to the identity. (from Evans/Zworski, Chapter 1)

Date: 17 January 2007.

WKB Approximation. Determination of quasimodes and assembly into a smooth approximate solution. Eikonal equation. Case of nonvanishing ∇V. Stable-unstable manifold theorem (statement only). Case of nondegenerate potential minimum. (from Dimassi/Sjöstrand, Chapters 2-3)

Pseudodifferential Operators. Quantization formulæ, composition of operators. S^k_δ symbol classes, semiclassical expansions, L^2 boundedness. Adjoints, products, inverses. Gårding inequalities. (from Evans/Zworski, Chapter 4)

Semiclassical Defect Measures. Construction, examples, positivity. Support, flow invariance, application to the damped wave equation on the torus. (from Evans/Zworski, Chapter 5)

Quantum Ergodicity. Weak Egorov theorem. Strong Weyl asymptotics (statement only). Schnirelmann's theorem. (from Evans/Zworski, Chapter 9)

Minor Topic: Quantum Mechanics (applied mathematics)

Basic Formalism. Bras, kets. Hermitian operators as observables, matrix elements, eigenstates. Expectation value, uncertainty principles, compatible observables, position and momentum operators, the need for unbounded operators. Measurement, Copenhagen interpretation, decoherence. Examples of Stern-Gerlach experiment, double pinhole experiment. (from Sakurai, Chapter 1)

Quantum Dynamics. Time evolution, Schrödinger and Heisenberg pictures. Simple harmonic oscillator, coherent states. One-dimensional WKB method, treatment of turning points using Airy functions, Bohr-Sommerfeld correspondence. Propagators, Feynman path integrals. Equivalence of the various formulations. Potentials and gauge transformations, Aharonov-Bohm effect. (from Sakurai, Chapter 2)

Approximation Techniques. Variational ground state estimates. Time-independent perturbation theory: non-degenerate and degenerate cases. Time dependent perturbation theory. Dyson series. Fermi's golden rule. (from Sakurai, Chapter 5)

References

[1] M. Christ *Euclidean Harmonic Analysis Notes for Mathematics 258.*
[2] M. Dimassi and J. Sjöstrand *Spectral Asymptotics in the Semiclassical Limit.*
[3] L.C. Evans and M. Zworski, *Lectures on Semiclassical Analysis.*
[4] L. Hörmander, *The Analysis of Linear Partial Differential Operators I: Distribution Theory and Fourier Analysis.*
[5] J.J. Sakurai *Modern Quantum Mechanics.*