Spring 2014

EMISSARY

Mathematical Sciences Research Institute

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Algebraic Topology

Chris Douglas, John Greenlees, and Haynes Miller

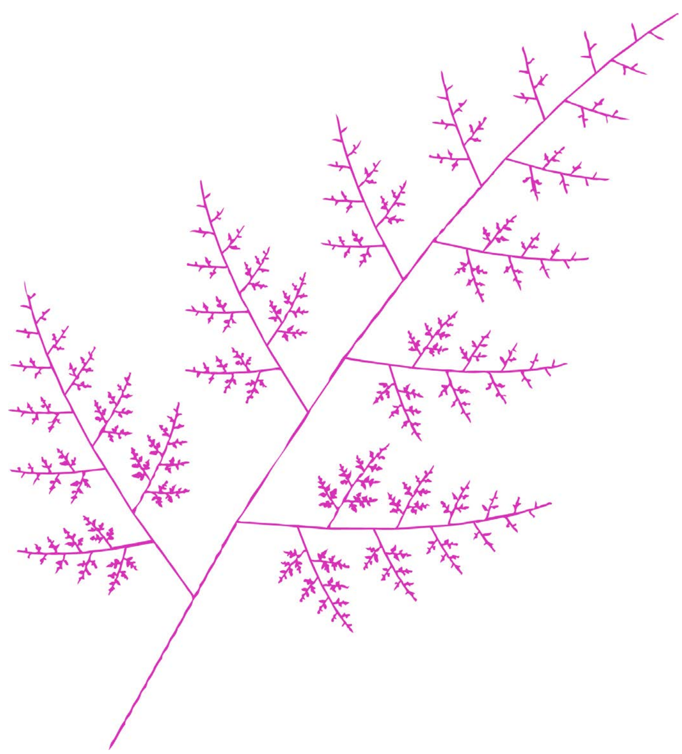
Algebraic topology is at a point of inflection today, and the Spring 2014 Algebraic Topology program at MSRI reflects the excitement of this moment. The introductory workshop, with more than 200 participants, provided careful introductions to the dominant themes leading up to this moment, giving an informative welcome to the many young researchers in the program and attending the conference. The workshop included a problem session, in which a selection of open problems was presented, some classical, some not-so-classical. Two series of weekly seminars — one on current research, another on open problems — continue to open vistas and improve contacts between diverse research groups. In the middle of the semester program there will be a further workshop, entitled "Reimagining the Foundations of Algebraic Topology." This conference will capture and disseminate the new spirit we see germinating today, linking higher category theory to the theory of manifolds, providing new approaches to the traditional bridge between topology and algebra provided by algebraic K-theory, and moving ever closer to a real understanding of the topological aspects of quantum field theory.

Ancient Roots

Algebraic topology was born around 1900 from Poincaré's desire to understand the orbits of the planets. The solar system presents a differential equation on a rather high-dimensional manifold, and Poincaré perceived that the geometry of a manifold would exercise a deep influence over solutions of differential equations defined on it. To understand this domain of large but finite dimensional spaces, he created a variety of invariants which have since become

(continued on page 4)

David Eisenbud



Spring branches: In the Berlekamp Garden at MSRI (top) and in Model Theory, with the Berkovich analytification of $\mathbb{P}_1(\mathbb{C})$. See page 8 for the Model Theory article.

Contents

Simons Endowment Campaign Surpasses $15 Million

Heike Friedman

With close to $15.5 million committed to MSRI's Endowment Fund, we are proud to announce the successful completion of the Simons Endowment Challenge Campaign. The Simons Foundation seeded the Fund with a $5 million gift naming the Eisenbud Professorships, and pledged an additional $5 million for the General Endowment Fund and Named Postdoctoral Fellowships if this amount could be matched. Thanks to the generous support of many of you, we exceeded this matching challenge by $500,000!

MSRI now has four named, semester-long postdoctoral positions with the latest dedicated to Elwyn Berlekamp (see page 7). The position was funded by a group of friends and former students of Elwyn's who jointly contributed more than $1 million to the fellowship. The endowed postdoctoral fellowships, named for Andrew Viterbi (two fellows), Craig Huneke, and Elwyn Berlekamp, total $3.1 million. In addition, we are delighted to announce the Roger Strauch Postdoctoral Fellowship, which is underwritten by a five-year donation to the Director's Fund.

While the Simons Challenge Campaign is completed, the MSRI Endowment Fund needs ongoing support to continue its growth. The Endowment helps to maintain fiscal stability to sustain and enhance excellence of programs and operations.

Underwriting a Postdoctoral Fellowship

MSRI offers two options to name a semester-long Postdoctoral Fellowship:

- Permanently with a gift to the Endowment Fund;
- Temporary with a multi-year gift commitment for a minimum of five years.

For more information on how to endow or underwrite a postdoctoral position, please contact Heike Friedman, Director of Development, hfriedman@msri.org, 510-643-6056.

THANK YOU TO ALL OUR DONORS TO THE SIMONS CHALLENGE ENDOWMENT CAMPAIGN!

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Four New Things

David Eisenbud, Director

The problem of writing a note for the Emissary is that there are too many interesting things happening at MSRI! Here's a sample of four from different domains.

Perfectoid Spaces

Hot Topics Workshops are always exciting events here, but this one was even more than usual. Some 250 people came to hear about the development that Peter Scholze started. His wonderful series of lectures was a high point of which people couldn't get enough, and there were many other fine talks, too. The subject (seen from a distance — the reader should know that it's not exactly my field!) is a direct connection between the two worlds of the theory of global fields: number fields, and curves over finite fields. The deep analogy between these worlds has been recognized for well over 100 years: a curve over a finite field k may be represented by a polynomial equation in one variable over $k(x)$, while a number field is an equation in one variable over the rational numbers (having the rational number line is "like" having a variable). The discovery by Scholze of a new *direct* bridge between these two worlds has the potential to change our thinking about why this analogy exists.

Public Outreach that Really Reaches Out

I have always felt that something was sad about the fact that a big mathematics lecture for the public might reach a thousand people, a football game gets an audience — much larger — of nearly 100,000. But the balance may be shifting! MSRI is supporting a YouTube star, Brady Haran, who has created the channel *Numberphile*. As of this writing the channel has over 850,000 subscribers, and the videos posted frequently have more than 500,000 views. Two of my favorites are "Pebbling a Chessboard" with Zvezdelina Stankova, and "Does John Conway hate his game of life?" — a Google search will get you either one.

How to be Family Friendly

The ideal way to come to MSRI as part of a semester-long or year-long program is to come for an extended period — a whole semester or a whole year. In my personal experience with sabbaticals, those long stays at wonderful places (MSRI was one of them — perhaps the reason I'm here now!) were far more productive later in the period than at the beginning. But coming to a distant place for a long period is a challenging feat of logistics, and particularly so for people with young families. The difficulty is particularly great for many young women, since women still play a disproportionate role in child care. MSRI strives to make it easier for these young families, and especially for women; but how could we help with this problem of transplanting families?

We are experimenting with a program that we hope will make a real difference: we have hired our first Family Services Coordinator, Sanjani Varkey (sanjani@msri.org). Sanjani is charged with knowing all that she can about the local schools and daycare possibilities, and will reach out to young families who are considering coming to MSRI. If you're in that position I hope you'll find this new service helpful!

Mosaic = Math+Art at MSRI's Academic Sponsors

Is your university one of MSRI's 98 Academic Sponsors? If it is, MSRI might fund an outreach event called Mosaic on or near your campus! It's a new benefit that we offer. You may already know of Bridges (www.bridgesmathart.org), headed by mathematician Reza Sarhangi, an organization that produces a large scale Math+Art conference each year (the next one is in Seoul, at the ICM). The same organization, with MSRI support, could organize a Mosaic conference — a sort of mini-Bridges conference — near you. If you're curious about this you can find more details, and even sign up to be considered, at www.mosaicmathart.org. For the moment, MSRI has the funds to sponsor six of these conferences per year.

Vesna Stojanoska

Viterbi Postdoc

Vesna Stojanoska, a member of the Algebraic Topology program, is the Spring 2014 Viterbi Endowed Postdoctoral Scholar. Vesna did her undergraduate studies at the American University in Blagoevgrad, Bulgaria. In the fall of 2006, she came to Northwestern University for her doctoral studies, which she completed under the supervision of Paul Goerss.

Vesna has done extensive work on duality in the relatively new field of derived algebraic geometry, which imports the flexibility of algebraic topology into geometry. In particular, Vesna has found a very explicit form of Serre duality for the derived moduli stack of elliptic curves. Since there is no simple statement for the underlying algebraic object, it was a major ratification of derived algebraic geometry that there is a much cleaner result in the seemingly more complex category.

The Viterbi Endowed Postdoctoral Scholarship is funded by a generous endowment from Dr. Andrew Viterbi, well known as the co-inventor of Code Division Multiple Access (CDMA) based digital cellular technology and the Viterbi decoding algorithm, used in many digital communication systems.

Algebraic Topology

(continued from page 1)

standard fare in undergraduate topology courses: the “Poincaré” or fundamental group, and what we now (following Emmy Noether) regard as the homology groups. In retrospect, the dominant themes of today’s practice of algebraic topology can already be seen in this early work, and the fundamental tension Poincaré confronted — discrete versus continuous — continues to inform the subject. The inflection point we find ourselves at today consists of a deepening of this creative interplay, with dramatic and quite unexpected connections coming into focus.

Topology has always provided a big tent, with a diversity of research fronts active simultaneously. Today is no exception, and by design the MSRI program brings together a large number of disparate research directions, linked by common ancestry and a common focus on the geometric structure of spaces, especially manifolds, and their hidden discrete nature that is revealed by combinatorial or algebraic invariants. Part of the excitement in the subject today is the richness of interaction between these far-flung branches of the algebraic topology family.

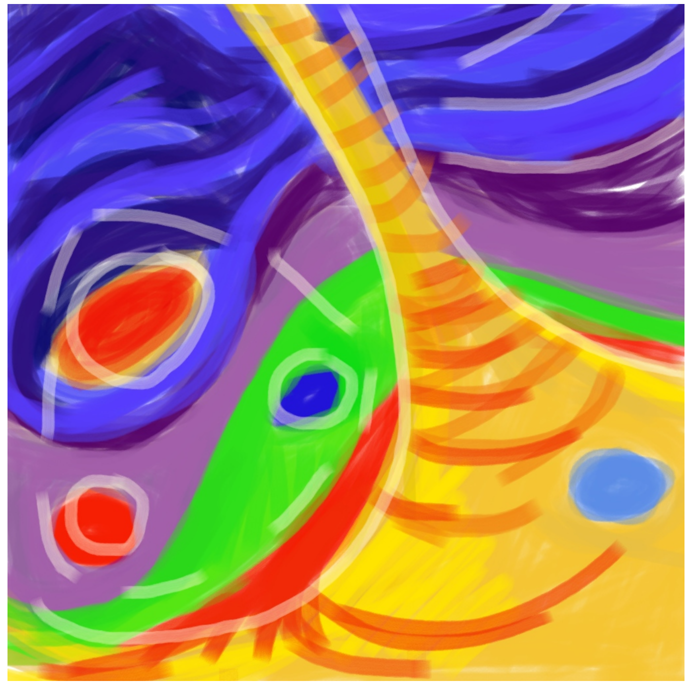
A Recent Victory: Resolution of Kervaire Invariant One

We will describe some of the work being done at MSRI, and a good place to begin is with the resolution of the Kervaire invariant question. This was a “nail problem,” left over from the great attack on the classification of manifolds and their automorphisms in the 1960s initiated by Kervaire and Milnor and carried forward by Browder, Novikov, Sullivan, Wall, and many others. Browder succeeded in giving it a purely homotopy-theoretic formulation, and it served as an organizing principle for much of the research in homotopy theory during the 1970s. But essentially no progress was made on it between the time of Browder’s work and its resolution, announced in 2009, by Mike Hill, Mike Hopkins, and Doug Ravenel. Their attack on this problem brought together many of the central research themes of the past twenty or thirty years, notably the chromatic, motivic, and equivariant modes of stable homotopy theory, as well as the theory of structured ring spectra. These topics form some of the central themes of the MSRI semester, and the insights gained from their interaction in the Hill–Hopkins–Ravenel proof provide important directions for future research.

Voevodsky’s solution of the Milnor conjecture opened the field of motivic homotopy theory, a broad contact zone between algebraic geometry and homotopy theory. Exotic variants of objects familiar in traditional homotopy theory have been found growing there and have solved more problems like the Milnor conjecture and — in work of Kriz, Hu, Dugger, Isaksen, and others — have shed new light on the classical homotopy theory picture.

Chromatic and Elliptic

The chromatic perspective stemmed from work of Daniel Quillen in the 1960s as amplified and interpreted by Jack Morava and others. It filters stable homotopy theory into strata, each of which exhibits a characteristic periodic behavior with wavelength increasing with depth (or, as it is better called, height). The surface layer consists of rational stable homotopy theory, which is essentially the theory of rational vector spaces. The next layer is dominated by topological K-theory, and much of the research of the second half of the twentieth century in homotopy theory was dedicated to understanding this stratum.



Chiral homology (constructed by considering families of disjoint discs) is an invariant that one can associate to an n-dimensional manifold and an algebra over the little n-discs operad.

Over the past twenty years the next chromatic layer has slowly been revealing itself. This is the domain of “elliptic cohomology,” an analogue of K-theory studied in a special case by Landweber, Ravenel, and Stong, and then in greater generality by Franke. Hopkins, with assists from Miller and Goerss, provided a lifting into homotopy theory of the arithmetic theory of elliptic curves. This was later reinterpreted by Lurie in a motivating example of his theory of “derived algebraic geometry.” The result of this approach is the creation of a new object, the spectrum TMF of topological modular forms, which provides a deep link between the classical theory of modular forms and the intricate structure of the second chromatic layer of stable homotopy theory. This connection has provided a rich and continuing vein of research — see the article on page 3 about Viterbi Postdoc Vesna Stojanoska, for example.

Just as K-theory served as a nexus of topology, geometry, analysis (in the form of index theory), and algebra (through its extension to algebraic K-theory), elliptic cohomology holds the promise of providing a meeting point in this century of topology, conformal field theory, and the arithmetic of elliptic curves. The connection with physics was stressed by Atiyah, Segal, and Witten, and has subsequently been pursued by Stolz and Teichner, by Bartels, Douglas, and Henriques, and by many others.

Elliptic cohomology provides an algebraic oasis in the arid formal expanse of the second chromatic layer. Work of Behrens and Lawson establishes analogous regions in other chromatic strata, providing a large family of analogues of TMF known as topological automorphic forms. How much of the full geography can be colonized from these oases remains to be seen; this is an exciting direction of current research.

Homotopicalization

A major trend in algebraic topology over the past few decades has been the effort to reverse the traditional flow of information, by creating topological (or, better, homotopy-theoretic) lifts of algebraic concepts. The spectra of stable homotopy theory can be thought of as "homotopicalizations" of abelian groups. Rings, especially commutative rings, lend themselves to this treatment as well, and huge effort has been devoted to understanding the resulting theory of "structured ring spectra." There are versions of Galois theory (Rognes), the Brauer group (Baker, Richter, Szymik), and Hochschild homology and cohomology (beginning with Bökstedt and carried on by many). The Hill–Hopkins–Ravenel work required the development of a theory of rings in equivariant stable homotopy theory.

In fact this lifting of ring theory has provided the most powerful method we have to compute algebraic K-theory, even the algebraic K-theory of discrete rings. This approach was initiated by Bökstedt, Hsiang, and Madsen, and carried on by Hesselholt, Madsen, and many others. It depends on a careful homotopical analysis of the naturality properties of the trace map from linear algebra, formalized in the theory of cyclotomic spectra. This theory has now been put on much better homotopy theoretic footing thanks to recent work of Blumberg and Mandell, and has been greatly clarified by application of some ideas originating in the Hill–Hopkins–Ravenel work by Angeltveit, Blumberg, Gerhardt, Hill, and Lawson.

It has been apparent since Waldhausen's seminal work on algebraic K-theory that the theory had interesting relations with the chromatic filtration; in very rough terms, it seems to increase chromatic height by 1. This has come to be known as the "red-shift" phenomenon: applying K-theory appears to increase wavelength. Work of Rognes, Ausoni, Dundas, and others has given more precise computational evidence for this phenomenon, which, however, remains one of the major mysteries of the subject. Not unrelated to this is the construction by Westerland of higher chromatic analogues of the J-homomorphism, a central actor in the K-theoretic stratum.

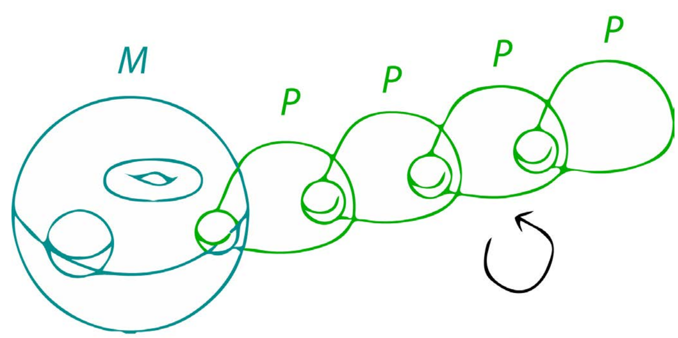
Calculus and Operads

Invariants of spaces X can often be expressed as the homotopy groups of some space $F(X)$, where F is a homotopy-preserving functor from spaces to spaces. Goodwillie created a "calculus" of such functors in which F is approximated by degree n functors and the layers in the tower are homogeneous and expressed in terms of the "derivatives" of F. This approximation method had its origins in geometric questions, but has proven extremely useful and has spawned (in work of Weiss and many others) a wide variety of analogues. It is now a central tool in much of homotopy theory. Johnson, Arone, Mahwowald, and Dwyer began the investigation of how it serves to connect stable and unstable phenomena, a line of investigation pursued by Behrens. Arone and Ching have proved "chain rules" for derivatives of composites, and constructed a descent scheme by which to reassemble the tower from its layers.

Closely connected with this is the theory of operads. Having its beginnings in topology (in work of Boardman, Vogt, and May), it subsequently enjoyed an algebraic "renaissance," and is now returning with new life to its geometric origins. Dwyer and Hess relate spaces of maps of operads to spaces of embeddings. Fresse relates the automorphism group of the little squares operad to the Grothendieck–Teichmüller group.

Cobordism Redux

Geometric topology has not been left behind. While surfaces were classified a century ago, the theory of their automorphism groups, or equivalently the theory of fiber bundles with two-dimensional fibers, continues to hold many mysteries. Much progress has been made on the study of characteristic classes of such bundles, under the banner of "cobordism categories." The "Mumford conjecture" was resolved by Madsen and Weiss by showing that the stable (under connected sum with tori) classifying space for surface bundles could be identified, not just rationally (as Mumford had conjectured) but integrally, with a space easily described within the context of stable homotopy theory. This surprising link, presaged by work of Tillmann, and the "scanning map" method of proof, has opened a broad area of research on stable automorphism groups of geometric objects. The surgery program has surfaced in a new guise: Galatius and Randal-Williams have combined this approach with work of Kreck to produce new results in higher dimensions. The issue of stability theorems has taken on renewed urgency, and the methodology of proving such results is being formalized in work of Wahl, Church, and others.



Stabilization by taking connected sum with a chosen manifold P often induces isomorphisms in the homology of diffeomorphism groups or mapping class groups, in a range increasing with the number of copies of P.

The cobordism category perspective relates naturally to the study of topological quantum field theories. Inspired by ideas from physics, this broad and very active field of contemporary research aims to produce new topological or smooth invariants of manifolds by providing a new linkage with purely algebraic structures.

The process of decomposing a manifold into pieces is modeled by higher category theory, in which morphisms are related to each other by higher "cells." Lurie's solution of the Baez–Dolan cobordism hypothesis has stoked the fires here. An important part of this picture is the construction of homology-like invariants specifically designed to capture properties of manifolds of a single dimension. These constructions, called chiral or factorization homology, are based on the classical study of configuration spaces. Made explicit by Lurie and by Andrade, they combine insights of Salvatore with representation-theoretic ideas of Beilinson and Drinfeld. Under the influence of Costello, Francis, and others, they offer the hope of organizing the many new invariants of interest to geometric topologists under a single rubric.

The use of higher category theory here is part of a larger paradigm shift underway in algebraic topology. Quillen's formalization of standard homotopy theoretic structures and processes by means of model categories can now be seen as an aspect of the theory of $(\infty, 1)$-categories. Originating in old work of Boardman and Vogt, these ideas were taken up by Joyal and Tierney and then embraced by Lurie as a model for homotopy theories. This broadening perspective is accompanied by non-negligible combinatorial demands, but results in a more conceptual approach to many questions by providing the means of avoiding arbitrary choices. During the MSRI workshop, work on an extension of this theory to an enriched setting was completed by Gepner and Haugseng. In another direction, work of Barwick, Schommer-Pries, Bergner, Rezk, and many others is focused on the axiomatics of the further extension to (∞, n)-categories, in which directionality is maintained into higher dimensions.

Seizing the Moment

The excitement of this moment of conjunction is reflected in the vigorous activity in the program this spring. Bringing together such an array of experts from across the world in such an outstanding research environment will undoubtedly bear diverse and unexpected fruit, within and beyond the field, thereby building the future by sustaining and inspiring a new generation of topologists.

Focus on the Scientist:
Bill Dwyer

William G. Dwyer, known as Bill to his fellow homotopy theorists, has had a profound influence on the evolution of homotopy theory and K-theory over the past four decades. Bill obtained his Ph.D. from MIT in 1973 under the direction of Dan Kan, after which he was a Gibbs Instructor at Yale for two years, then at the Institute for Advanced Study for one year. He returned to Yale as an assistant professor in 1976, staying until 1980, when he moved to Notre Dame, where he is now professor emeritus.

Bill collaborated frequently with his former advisor, writing a total of 32 joint papers, many of which have had a lasting influence on algebraic topology. In a series of landmark papers published in 1980, Bill and Dan developed the theory of function complexes in categories with weak equivalences, laying the foundations of modern abstract homotopy theory.

Bill Dwyer

Bill's interests and contributions extend far beyond the borders of pure homotopy theory. In groundbreaking work in the early 1980s, Bill and Eric Friedlander introduced étale K-theory, a twisted generalized cohomology theory on the étale homotopy type of a Noetherian scheme, essentially constructed from topological K-theory. Étale K-theory leads to a geometric reformulation of the famous Quillen–Lichtenbaum conjecture, which allows this conjecture to be interpreted in terms of either homotopy types of K-theory spectra or the cohomology of general linear groups.

In another remarkable example of his work at the intersection of homotopy theory and other fields, Bill collaborated with Clarence Wilkerson in the early 1990s on founding and elaborating the theory of p-compact groups, a homotopy-theoretic rendering of the theory of compact Lie groups, encompassing maximal tori and Weyl groups. For any prime number p, a p-compact group is a p-complete space whose loop space has finite mod p cohomology. The p-completed classifying space of a Lie group whose group of components is a finite p-group is always a p-compact group, but there are also important, "exotic" p-compact groups that do not arise in this manner. That Bill was invited to lecture on p-compact groups at the 1998 ICM in Berlin attests to the importance of this new theory in algebraic topology.

More recently, Bill has also applied homotopy theory to commutative algebra and group cohomology. In joint work with John Greenlees and Srikanth Iyengar, Bill developed a remarkable and influential generalized homotopical duality theory, incorporating Poincaré duality for manifolds, Gorenstein duality for commutative rings, Benson–Carlson duality in group cohomology, and Gross–Hopkins duality in stable homotopy theory.

Bill is greatly appreciated by his colleagues not only for his substantial mathematical contributions but also for his warmth, congeniality, and wonderfully quick and dry sense of humor. It is an honor and pleasure for the members of the Algebraic Topology program to be able to count on Bill's presence throughout the semester at MSRI.

— Kathryn Hess

The Berlekamp Postdoctoral Fellowship

David Eisenbud

I'm quite delighted to announce the completion of a fundraising campaign to endow a Postdoctoral Fellowship in honor of Elwyn Berlekamp! Indeed, so many of Elwyn's friends and admirers were eager to contribute to the campaign that we were able to go well over our goal and raise $1,000,000 for the endowment, providing for a little fund for Fellows to use in addition to the basic stipend.

Elwyn's careers, in computer science and engineering, in mathematics and in business, are of great distinction. As all his friends know, Elwyn loves mathematical puzzles and problems; as an undergraduate, he was one of the five top scorers in the notoriously difficult Putnam competition. After completing a Ph.D. in Electrical Engineering from MIT in 1964, Elwyn held positions in Berkeley, JPL, MIT and Bell Labs. Although he is now professor emeritus in the math department at Berkeley, he was at one time the chair of computer science there.

In computer science and information theory, Elwyn is famous for his algorithms in coding theory and for the factorization of polynomials. In mathematics, his best-known work is on combinatorial game theory, partly disseminated in his four-volume work "Winning Ways" with John H. Conway and Richard Guy.

One of his important accomplishments in game theory was his analysis of positions in the endgame of Go. He demonstrated the effectiveness of his theory by setting up a plausible endgame position from which he beat one of the Japanese champions of the game, after which he set up the same position, reversed the board, and beat the master a second time. And again and again, for a total of seven consecutive wins. He also invented a variation of the game called "Coupon Go," which is closer to the elegant mathematical theories. This has attracted the attention of both mathematicians and several world-class professional Go players.

Elwyn's love of game strategy extends to everyday life as well: I have always been impressed by the fact that once, in a meeting of Berkeley's computer science department when someone proposed a motion of no-confidence against the chair, Elwyn seconded it, and amid general laughter the motion was dropped. The chair was . . . Elwyn!

Elwyn's father was a minister, and one sees the father's influence on the son in a strong and consistent ideal of service to the greater good, abundantly clear in Elwyn's commitment to MSRI among other institutions. In fact, his engagement with MSRI began even before there was an MSRI. Elwyn recounts going along for a meeting with the chancellor to convince him of an aspect of MSRI's structure and finding an easy task: the chancellor began the meeting by announcing that he approved the arrangement. (Years later, Elwyn taught me an important lesson of negotiation: once you have agreement, change the subject! I don't know how that conversation in the chancellor's office continued.)

Meeting Elwyn

Elwyn was Chair of MSRI's Board in 1996 when I applied to become director. He took his role extraordinarily seriously: to make sure that I was OK, he made a visit to my home near Boston (I was teaching at Brandeis at the time). I invited my colleague and mentor David Buchsbaum to join us for brunch, to bolster my team. I remember that after Elwyn left, Buchsbaum commented that he would worry about collaborating with someone quite so intense as Berlekamp!

After Elwyn hired me, the intensity turned out to be very positive. Elwyn mentored and coached me in what was, for me, an extraordinary experience of growth and learning. He introduced me to a wide and useful acquaintance and liberally allowed me to use his connections. During long car rides, I learned a great deal about the history of MSRI, in which he'd been very engaged, and the many personalities that had played a role. I count myself most fortunate to have had as mentors Saunders MacLane, my Ph.D. advisor; David Buchsbaum, my postdoctoral mentor, and longtime friend and collaborator; and finally Elwyn, who taught me so much and helped me in the transition to my role at MSRI, and whose friendship and encouragement has meant a great deal to me.

For all these reasons, it is a real pleasure to have established the Berlekamp Postdoctoral Fellowship at MSRI in addition to the Berlekamp Garden, created in 2006. May the Fellows go on to do great work in mathematics and for the mathematics profession!

2013–14 Chancellor's Professor

The UC Berkeley Chancellor's Professorship award carries a purse of $50,000 and is open to nominees from MSRI only. Chancellor's Professors must be top researchers and must also be known for excellent teaching.

The 2013–14 Chancellor's Professor is **Peter Scholze** of Universität Bonn. Last fall, Peter gave a series of lectures at the summer graduate school on New Geometric Techniques in Number Theory, and this past February, he gave a series of Hot Topics lectures on Perfectoid Spaces and their Applications.

Three Recent Applications of Model Theory

Rahim Moosa

This spring's Model Theory, Arithmetic Geometry, and Number Theory program is centered on recent interactions between model theory (a branch of mathematical logic) and other parts of mathematics. To give some idea of what these interactions are, I will discuss three particular examples of applications of model theory: to Berkovich spaces, to approximate subgroups, and to the André–Oort Conjecture for $\mathbb{C}^n$. Except for some concluding remarks on model theory, I will say almost nothing about the techniques and ideas that are behind the proofs of these theorems, and only hope that the interested reader will pursue his or her own further investigations.

Each of the applications I will discuss was the subject of tutorials in the introductory workshop of our program as well as Séminaire Bourbaki articles. The tutorials were by Martin Hils, Lou van den Dries, and Kobi Peterzil, respectively, and the corresponding articles are by Antoine Ducros, Lou van den Dries, and Thomas Scanlon. I have relied heavily on these sources, and it is to them that I direct the reader for further expository details.

(Videos and supplemental materials from all the tutorials from the introductory workshop are available on the MSRI web page www.msri.org/workshops/688. Follow the links in the workshop schedule at the bottom of the page.)

Berkovich Spaces

In a recent manuscript entitled "Non-Archimedean tame topology and stably dominated types," Hrushovski and Loeser use model theory to develop a framework for studying the analytic geometry associated to an algebraic variety over a non-Archimedean valued field. As a consequence they deduce several new results on Berkovich spaces.

Fix a complete non-Archimedean absolute valued field $(K, |\cdot|)$. *Non-Archimedean* refers to the fact that $|\cdot| : K \to \mathbb{R}_{\geqslant 0}$ satisfies the *ultrametric inequality*

$$|a+b| \leqslant \max\{|a|, |b|\}$$

and *complete* means with respect to the induced metric. The prototypical examples are: the field of p-adic numbers $\mathbb{Q}_p$, the completion of the algebraic closure of $\mathbb{Q}_p$, and the Laurent series fields $k((t))$. Now consider an algebraic variety V over K. In analogy with real or complex algebraic varieties, one would like to use the metric structure on K to consider $V(K)$ from the point of view of analytic geometry. The problem is that the topology that $|\cdot|$ induces on $V(K)$ is totally disconnected. In the early nineties, Berkovich proposed to resolve this deficiency by considering an enriched space V_K^{an} whose points are pairs (x, ν) where x is a scheme-theoretic point of V and $\nu : K(x) \to \mathbb{R}_{\geqslant 0}$ is an absolute value extending that of K. More concretely, in the case when V is affine, V_K^{an} can be canonically identified with the set of *multiplicative seminorms* on the co-ordinate ring $K[V]$; that is, multiplicative maps $\nu : K[V] \to \mathbb{R}_{\geqslant 0}$ that extend the absolute value on K and satisfy the ultrametric inequality. The topology induced on V_K^{an} from the product topology on $\mathbb{R}^{K[V]}$ is then locally path connected and locally compact.

Berkovich spaces have proved to have many and diverse applications. They have led to the development of p-adic analogues of classical notions from complex analysis including spectral theory, harmonic analysis, equidistribution, and dynamics. There have been applications to the Langlands program in arithmetic geometry via the development of étale cohomology of analytic spaces. Finally, by endowing a given ground field with the trivial absolute value (which, note, is complete and non-Archimedean), Berkovich spaces have also been useful in general algebraic geometry.

Hrushovski and Loeser use model theory to show that Berkovich spaces exhibit very tame topological behavior, generalizing and strengthening what was known before. Here are some of their results.

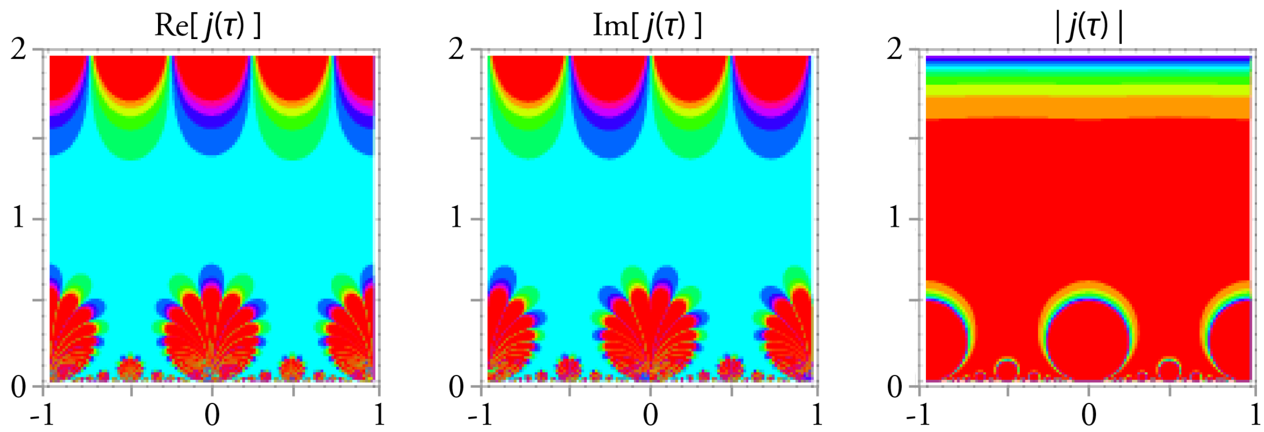
Theorem 1 (Hrushovski, Loeser). *Suppose* V *is a quasi-projective variety over* K*. Then:*

1. V_K^{an} *admits a strong deformation retraction to a closed subspace that is homeomorphic to a finite simplicial complex,*
2. V_K^{an} *is locally contractible, and*
3. *given a morphism* $f : V \to W$ *to an algebraic variety* W *over* K*, among the fibres of* $f^{an} : V_K^{an} \to W_K^{an}$ *there are only finitely many homotopy types.*

Approximate Groups

Given a positive integer K, a K-*approximate group* is a finite subset X of a group G such that $1 \in X$, $X^{-1} = X$, and $X^2 := \{xy : x, y \in X\}$ is covered by K left translates of X. This is supposed to say that X is almost closed under multiplication; so one should think of K as being fixed and of $|X|$ as being large compared to K. A 1-approximate group is a subgroup, and an easy example of a 2-approximate group that is not a subgroup is the set $\{-N, \ldots, N\}$ in $\mathbb{Z}$, for any $N > 0$. But the interest here is really when G is not commutative; approximate subgroups were introduced by Tao while studying the extension of additive combinatorics to the non-commutative setting.

In his 2012 paper entitled "Stable group theory and approximate subgroups," Hrushovski studies the structure of K-approximate groups as the cardinality $|X|$ goes to infinity by applying model-theoretic techniques to the logical limits (that is, *ultraproducts*) of sequences of K-approximate groups. His main achievement is to model such a limit of approximate groups by a compact neighborhood of the identity in a Lie group. This is reminiscent of the proof of Gromov's theorem on groups of polynomial growth; indeed, one of the striking applications of Hrushovski's work is a strengthening (and new proof) of Gromov's theorem. Another application is an extension of the Freiman–Ruzsa theorem to the non-commutative setting: in a group of finite exponent, every K-approximate group is *commensurable* to an actual subgroup, commensurable here in the sense that each is contained in finitely many left translates of the other, where the number of translates is bounded in terms of K. But the most celebrated application is the theorem of Breuillard, Green,



Contours of the j-function.

and Tao saying roughly that approximate groups are in general controlled by nilpotent groups. This appears in their 2012 paper "The structure of approximate groups," where they also give alternative proofs of some of Hrushovski's results. Here is a weak version of their theorem that is simple to state.

Theorem 2 (Breuillard, Green, Tao). *Given $K \geqslant 1$, there exists $L \geqslant 1$ such that for any K-approximate group $X \subseteq G$ there is a finite set $Y \subseteq \langle X \rangle$ such that X is covered by L left translates of Y, Y is covered by L left translates of X, and $\langle Y \rangle$ has a nilpotent subgroup of finite index.*

Among the applications of this theorem is a finitary version of Gromov's theorem and a generalized Margulis lemma that was conjectured by Gromov.

André–Oort for $\mathbb{C}^n$

Model theory's first spectacular application to Diophantine geometry was Hrushovski's solution in the early nineties to the function-field Mordell–Lang conjecture in all characteristics. This was one of the central themes of the 1998 MSRI program on the model theory of fields. In recent years there has been another round of Diophantine applications, this time to the André–Oort conjecture, in which model theory plays a very different role. The model theory behind these latest interactions stems from the 2006 paper of Pila and Wilkie that used model theory to count rational points on a certain class of subsets of $\mathbb{R}^n$ with tame topological properties. Following a general strategy proposed by Zannier, there are now a number of applications of this result in various directions. I will focus on what is possibly the most striking one thus far: Pila's solution to the André–Oort conjecture for $\mathbb{C}^n$.

Recall that to each point τ in the upper half plane $\mathbb{H} := \{z \in \mathbb{C} : \operatorname{Im}(z) > 0\}$, we can associate the elliptic curve $E_\tau := \mathbb{C}/(\mathbb{Z} + \mathbb{Z}\tau)$. The elliptic curve E_τ is said to have *complex multiplication* if its endomorphism ring is strictly bigger than $\mathbb{Z}$, which is equivalent to τ belonging to an imaginary quadratic extension of $\mathbb{Q}$. Now, there is a holomorphic surjection $j : \mathbb{H} \to \mathbb{C}$ with the property that $j(\tau_1) = j(\tau_2)$ if and only if E_{τ_1} and E_{τ_2} are isomorphic. We are interested in the affine varieties $X \subseteq \mathbb{A}^n_{\mathbb{C}}$ which have a Zariski dense set of points of the form $\big(j(\tau_1), \dots, j(\tau_n)\big)$ where each E_{τ_i} has complex multiplication. One thinks of the set of these points, called *special points*, as being in some way arithmetical, roughly analogous to the set of torsion points on a semiabelian variety. It is a fact that the special points are Zariski dense in $\mathbb{A}^n_{\mathbb{C}}$, so affine space itself gives us examples of such varieties X. More interesting examples are obtained by considering the Hecke correspondences

$$T_N := \{\big(j(\tau), j(N\tau)\big) : \tau \in \mathbb{H}\}$$

for each positive integer N. It turns out that T_N is an algebraic curve in $\mathbb{A}^2$. It has a Zariski dense set of special points since if τ is in a quadratic imaginary extension of $\mathbb{Q}$, then so is $N\tau$. The André–Oort conjecture for $\mathbb{C}^n$, proved by Pila in 2011, says that all examples come from the above two types. More precisely:

Theorem 3 (Pila). *Suppose $X \subseteq \mathbb{A}^n_{\mathbb{C}}$ is an irreducible subvariety containing a Zariski dense set of special points. Then X is an irreducible component of an intersection of varieties of the form:*

- $S_{i,\tau} := \{(z_1, \dots, z_n) : z_i = j(\tau)\}$, *where E_τ has complex multiplication, and*
- $T_{i,j,N} := \{(z_1, \dots, z_n) : (z_i, z_j) \in T_N\}$, *where $N > 0$.*

And Behind Them All: Model Theory

To the reader unfamiliar with model theory it may be surprising that the above theorems are all applications of a single subject and, at that, a branch of mathematical logic. In fact, model theory often plays the role of recognizing, formalizing, and facilitating analogies between different mathematical settings. In this final section I would like to say a few words about what model theory is.

The fundamental notion in model theory is that of a structure. A *structure* consists of an underlying set M together with a set of distinguished subsets of various Cartesian powers of M called the *basic relations*. It is assumed that equality is a basic (binary) relation in every structure. One could also allow *basic functions* from various Cartesian powers of M to M, but by replacing them with their graphs we can restrict to relational structures. For example, a ring can be viewed as a structure where the underlying set is the set of elements of the ring and there are, besides equality, two basic relations: the ternary relations given by the graphs of addition and multiplication. If the ring also admits an ordering that we are interested in, then we can consider the new structure where we add the ordering as another basic binary relation. The *definable sets* of a structure are those subsets of Cartesian powers of M that are obtained from the basic relations in finitely many steps using the following operations: intersection, union, complement, Cartesian product, image under a coordinate projection, and fibre of a coordinate projection. When $(R, +, \times)$ is a commutative unitary ring, for example, one sees immediately that if $f_1, \ldots, f_\ell$ are polynomials in $R[x_1, \ldots, x_n]$, then their set of common zeros in R^n is definable. Hence the Zariski constructible subsets of R^n are all definable. It is an important fact that if R is an algebraically closed field, then these are the only definable sets. This is *quantifier elimination* for algebraically closed fields, or, equivalently, Chevalley's theorem that over an algebraically closed field the projection of a constructible set is again constructible.

In any case, given a structure, model theory is concerned with this associated class of definable sets. Of course, starting with an arbitrary structure one cannot expect to say much. A key aspect is the isolation of *tameness* conditions under which the definable sets are in some way tractable. For example, algebraically closed fields are *strongly minimal* because the definable subsets of the field itself are all uniformly finite or cofinite. Strongly minimal structures admit a very well-behaved notion of *dimension* for definable sets. Real closed fields, on the other hand, display a different kind of tameness: they are *o-minimal* in that every definable subset of the line is a finite union of intervals and points — and this too leads to a (differently) well-behaved notion of dimension on the Cartesian powers. Strong minimality and o-minimality are only at the beginning of extensive hierarchies of tameness notions. Algebraically closed valued fields, for example, with their strongly minimal residue field and o-minimal value group, involve a certain comingling of the two.

Behind Pila's proof of the André–Oort conjecture for $\mathbb{C}^n$ is the definability of the j-function (restricted to a suitable fundamental set) in some o-minimal structure on the reals and the Pila–Wilkie theorem on counting rational points on definable sets in such structures. The theorems of Hrushovki and Loeser on Berkovich spaces use the tameness of definable sets in algebraically closed valued fields. The structure that lies behind the work of Hrushovski and that of Breuillard, Green and Tao on approximate groups is an ultraproduct of K-approximate groups. In each of the applications that I have discussed, the model theoretic techniques and ideas that are brought to bear on the problem are quite specialized, and it would be misleading to suggest some underlying or overarching principle. Nevertheless, they all stem from the perspective that model theory offers, and it is this perspective that brings together the themes, and participants, of our program.

Focus on the Scientist:

Zoé Chatzidakis

Zoé Chatzidakis is a French mathematician who came to the U.S. for graduate study. She received her Ph.D. at Yale under the direction of A. Macintyre, after which she worked at Princeton for several years before returning to Paris. She is now a senior CNRS researcher working at Université Paris Diderot and École Normale Supérieure in Paris. She is currently visiting MSRI as a research professor in the program on Model Theory, Arithmetic Geometry, and Number Theory.

Zoé Chatzidakis

Zoé is a model theorist whose contributions range from pure model theory to applications to problems in algebra and number theory. A fundamental work by Zoé (in collaboration with L. van den Dries and A. Macintyre) deals with the study of definable sets over finite fields. Their main result provides uniform estimates for such sets. A nice consequence is that there is no first order formula in the language of rings, which defines $\mathbf{F}_q$ uniformly in every finite field $\mathbf{F}_{q^2}$. Such estimates are fundamental to the understanding of asymptotic properties of finite fields. For instance, they were recently used by T. Tao in his proof of an algebraic version of the Szemerédi regularity lemma.

Zoé also made important contributions to the model theoretic study of fields with an automorphism. In particular, in a series of two papers, the first with E. Hrushovski and the second joined by a third author Y. Peterzil, she established basic trichotomy theorems. These results were used in a fundamental way in the celebrated work of Hrushovski on the Manin–Mumford conjecture. Another spectacular application of the model theoretic study of difference fields, due to Chatzidakis and Hrushovski, concerns the descent properties of algebraic dynamical systems. Zoé's fields of interest also include Diophantine geometry: together with D. Ghioca, D. Masser and G. Maurin, she recently formulated and proved a function-field analogue of the Zilber–Pink conjecture on unlikely intersections.

Amongst Zoé's contributions to abstract model theory, her recent work on the canonical base property gives an abstract version of a phenomena first found in compact complex manifolds with far reaching applications to differential algebra, difference algebra, and algebraic dynamics.

Zoé is the recipient of the 2013 Leconte Prize of the French Academy of Sciences and an invited speaker at the 2014 ICM in Seoul in the Logic and Foundations section.

— François Loeser

The Pleasure of What's Not on the Test

Anne Brooks Pfister

Last fall, MSRI and Berkeley City College (BCC) debuted a new lecture series, "Not on the Test: The Pleasures and Uses of Mathematics." Held in BCC's auditorium in downtown Berkeley, the series of six free, public talks is made possible through generous funding from the Simons Foundation. You can find videos of the lectures (except Tony DeRose's), made by BCC, by searching on the web for "P-Span" plus the speaker's name.

In the first talk, **Math in the Movies**, Tony DeRose, senior scientist and lead of the research group at Pixar Animation Studios, wowed an overflow audience with clips from Pixar films including *Finding Nemo* and *Ratatouille* that demonstrated the mathematical principles that were applied in the movie-making process.

Keith Devlin's presentation, **Video Games for Mathematics**, showed how casual games that provide representations of mathematics enable children (and adults) to learn basic mathematics by "playing" — in the same way music is learned by learning to play the piano. Professor Devlin is a mathematician at Stanford University and also known as "the Math Guy" on National Public Radio.

Inez Fung spoke about **Verifying Greenhouse Gas Emissions**. Addressing the highly topical question, "How well do we know that greenhouse gas emission targets are being met?", she showed how data assimilation techniques are used to merge observations with models to verify target levels. Dr. Fung, a professor of atmospheric science at UC Berkeley, is a contributing author to the Assessment Reports of the Intergovernmental Panel on Climate Change (IPCC), the UN-based scientific body that shared the 2007 Nobel Peace Prize with Vice President Al Gore.

Ge Wang brought together **Music, Computing, People** before a rapt audience that filled the venue. Dr. Wang's presentation explored the transformative possibilities of combining music with computing, art, and technology in an emerging dimension where people around the world interact through social music apps. One listener remarked that the presentation made "wonderful connections between tech and humanity." Dr. Wang is an assistant professor at Stanford University in the Center for Computer Research in Music and Acoustics and is the founding director of the Stanford Laptop Orchestra and of the Stanford Mobile Phone Orchestra. The event was co-presented with the Simons Institute for the Theory of Computing.


Ge Wang (l) and Philip Sabes (r), two speakers who will let you know what's not on the test.

Eugenie Scott confronted **Science Denialism**, describing how, outside of scientific circles, rhetoric and factual anomalies are used to place science — in particular, evolution and global warming — under attack for ideological reasons. Dr. Scott, the executive director of the National Center for Science Education, Inc., is the author of *Evolution vs Creationism: An Introduction* and co-editor with Glenn Branch of *Not in Our Classrooms: Why Intelligent Design is Wrong for Our Schools*.

The final talk of the series, Philip Sabes' presentation on **Brain-Computer Interfaces**, will take place on April 9. He will discuss how machine interfaces offer the promise of helping disabled patients to control prosthetic limbs and computer interfaces directly from their brain. Dr. Sabes is a Professor of Physiology at the University of California, San Francisco, and the director of the UCSF Swartz Center for Theoretical Neurobiology.

The "Not on the Test" lecture series will continue during the next academic year 2014–15. Visit www.msri.org *in September to see the line-up of distinguished speakers and intriguing topics related to math, culture, and society.*

Focus on the Scientist: James Freitag

James Freitag

James Freitag is a member of the MSRI Program in Model Theory, Arithmetic Geometry, and Number Theory and is currently a National Science Foundation postdoctoral fellow at Berkeley working with Thomas Scanlon. Jim completed his undergraduate study at the University of Illinois at Urbana-Champaign in 2006, then earned a masters in industrial mathematics at Michigan State in 2007 before coming to the University of Illinois at Chicago where he earned his Ph.D. in 2012 under the direction of David Marker.

Most of Jim's research is focused on applying tools from model theory, a branch of mathematical logic, to differential algebra and differential algebraic geometry. Applying ideas from the model theory of groups, Jim showed that the non-commutative almost simple linear differential algebraic groups are equal to their commutator subgroups, a conjecture made by differential algebraists.

Strauch Postdoc

Pierre Simon, a member of the Model Theory, Arithmetic Geometry and Number Theory program, is the Spring 2014 Strauch Endowed Postdoctoral Scholar. Pierre did his undergraduate studies at École Normale Supérieure in Paris (France) and obtained the agrégation in 2007. He obtained his Ph.D. in 2011 under the supervision of Elisabeth Bouscaren at the University of Paris-Sud (Orsay, France).

Pierre Simon

Pierre works in Pure Model Theory, and more particularly on NIP theories, where he obtained some fundamental results and is the author of a monograph. In 2012 he received two prizes for his thesis work: the Perrisin-Pirasset/Schneider prize from the Chancellerie des Universités de Paris (awarded each year to a thesis in mathematics defended in the Paris region), and the Sacks prize (awarded by the ASL to the year's best thesis in logic). He was a postdoctoral fellow at the Hebrew University until December 2013 and started a researcher position at the CNRS in January 2014.

Roger Strauch is Chairman of The Roda Group, a seed stage venture capital group based in Berkeley, California. His firm, co-founded in 1997 with Dan Miller, provides entrepreneurs the resources, environment, and guidance to launch and grow their high technology businesses. The Roda Group is one of the main investors in Solazyme, a renewable oil and bioproducts company and the leader in algal biotechnology.

Mr. Strauch is a member of the Engineering Dean's College Advisory Boards of the University of California at Berkeley and Cornell University. He is the recipient of the 2002 Wheeler Oak Meritorious Award from the University of California at Berkeley. Mr. Strauch is also currently the chair of MSRI's Board of Trustees, on which he has served for more than 15 years.

Forthcoming Workshops

May 12, 2014–May 16, 2014: *Model Theory in Geometry and Arithmetic*, organized by Raf Cluckers, Jonathan Pila (Lead), Thomas Scanlon

June 16, 2014–June 27, 2014: *Dispersive Partial Differential Equations*, organized by Natasa Pavlovic, Nikolaos Tzirakis

June 21, 2014–August 03, 2014: *MSRI-UP 2014: Arithmetic Aspects of Elementary Functions*, organized by Duane Cooper, Ricardo Cortez, Herbert Medina (Lead), Ivelisse M. Rubio, Suzanne Weekes. Lecturer, Victor Moll

June 23, 2014–July 04, 2014: *Séminaire de Mathématiques Supérieures 2014: Counting Arithmetic Objects*, organized by Henri Darmon, Andrew Granville, Benedict Gross. (Montréal, Canada)

June 29, 2014–July 19, 2014: *IAS/PCMI 2014: Mathematics and Materials*, organized by Mark Bowick, David Kinderlehrer, Govind Menon, Charles Radin. (Park City, Utah)

June 30, 2014–July 11, 2014: *Algebraic Topology Summer Graduate School*, organized by Jose Cantarero-Lopez, Michael Hill. (Guanajuato, Mexico)

July 07, 2014–July 18, 2014: *Stochastic Partial Differential Equations*, organized by Yuri Bakhtin, Ivan Corwin (Lead), James Nolen

July 28, 2014–August 08, 2014: *Geometry and Analysis*, organized by Hans-Joachim Hein, Aaron Naber (Lead)

August 14, 2014–August 15, 2014: *Connections for Women: New Geometric Methods in Number Theory and Automorphic Forms*, organized by Wen-Ch'ing Li, Elena Mantovan (Lead), Sophie Morel, Ramdorai Sujatha

August 18, 2014–August 22, 2014: *Introductory Workshop: New Geometric Methods in Number Theory and Automorphic Forms*, organized by Laurent Berger, Ariane Mezard, Akshay Venkatesh (Lead), Shou-Wu Zhang

August 28, 2014–August 29, 2014: *Connections for Women: Geometric Representation Theory*, organized by Monica Vazirani (Lead), Eva Viehmann

For more information about any of these workshops as well as a full list of all upcoming workshops and programs, please see www.msri.org/scientific.

Call for Membership Applications

MSRI invites membership applications for the 2015–2016 academic year in these positions:

Research Professors	by October 1, 2014
Research Members	by December 1, 2014
Postdoctoral Fellows	by December 1, 2014

In the academic year 2015–2016, the research programs are:

New Challenges in PDE: Deterministic Dynamics and Randomness in High and Infinite Dimensional Systems, Aug 17–Dec 18, 2015
Organized by Kay Kirkpatrick, Yvan Martel, Jonathan Mattingly, Andrea Nahmod, Pierre Raphael, Luc Rey-Bellet , Gigliola Staffilani, Daniel Tataru

Differential Geometry, Jan 11–May 20, 2016
Organized by Tobias Colding, Simon Donaldson, John Lott, Natasa Sesum, Gian Tian, Jeff Viaclovsky

MSRI uses **MathJobs** to process applications for its positions. Interested candidates must apply online at www.mathjobs.org after August 1, 2014. For more information about any of the programs, please see www.msri.org/scientific/programs.

Four New Staff and Consultants Join MSRI

Kirsten Bohl started in January as MSRI's new Outreach Producer, a position funded through a three-year grant for national outreach about mathematics through the Simons Foundation. She brings experience in higher education and K–12 development, communications, and event planning. She is enthusiastic about transforming public attitudes about math. New to the Bay Area, she is enjoying exploring the natural world on the western edge of the continent.

Heike Friedman joined the MSRI staff in January as Director of Development. In addition to her seven years experience as Development Director at Tehiyah Day School in El Cerrito, Heike has a background in journalism and public relations. A German native, Heike received her M.A. in German literature from the Ruhr-Universität Bochum. She loves traveling, especially exploring big cities and dragging her husband and teenage daughter to art museums all over the world. At home, she enjoys the "great indoors" of her Berkeley home, cooking, knitting, and watching old movies and new theater plays.

Peter Trapa, the new National Association of Math Circles Director, earned his Ph.D. from MIT and held postdoctoral positions at Harvard and the Institute for Advanced Study in Princeton before joining the faculty at the University of Utah, where he is now professor and chair. He has a longstanding interest and appreciation for working with mathematically talented kids, beginning in his days as a graduate student (where he had the good fortune of tutoring several phenomenal middle and high school students) and continuing through his work with the Utah Math Circle (where he served as coordinator for the past decade or so).

Sanjani Varkey has been the Family Services Consultant since December 2013. Before coming to MSRI, she spent seven years working in the field of public health in India and South Africa, doing research, advocacy, training and community organization. This was followed by a decade of being the full-time parent to her daughters. She grew up in India, and now considers Berkeley home, after stops in Pachod (a village in India), Johannesburg, Grass Lake (a village in Michigan) and Carlsbad, CA. She has an M.A. in social work from TISS, India, and a Master of Public Health from the University of Michigan.

Kirsten Bohl

Heike Friedman

Peter Trapa

Sanjani Varkey

Call for Proposals

All proposals can be submitted to the Director or Deputy Director or any member of the Scientific Advisory Committee with a copy to proposals@msri.org. For detailed information, please see the website www.msri.org.

Thematic Programs

Letters of intent and proposals for semester or year long programs at the Mathematical Sciences Research Institute (MSRI) are considered in the fall and winter each year, and should be submitted preferably by **October 15** or **December 15**. Organizers are advised that a lead time of several years is required, and are encouraged to submit a letter of intent prior to preparing a pre-proposal. For complete details see http://tinyurl.com/msri-progprop.

Hot Topics Workshops

Each year MSRI runs a week-long workshop on some area of intense mathematical activity chosen the previous fall. Proposals for such workshops should be submitted by **October 15** or **December 15**. See http://tinyurl.com/msri-htw.

Summer Graduate Schools

Every summer MSRI organizes four 2-week long summer graduate workshops, most of which are held at MSRI. To be considered for the summer of year n, proposals should be submitted by **October 15** or **December 15** of year $n-2$. See http://tinyurl.com/msri-sgs.

Focus on the Scientist:
John Greenlees

John Greenlees, a member of the Algebraic Topology program running at MSRI during the spring of 2014, is a leading expert in equivariant stable homotopy theory and for homotopical algebra over structured ring spectra. Equivariant homotopy theory studies spaces with symmetries, up to continuous deformation. Also allowing suspension by the spheres of orthogonal representations adds a strong algebraic flavor; the resulting objects are geometric incarnations of algebraic structures such as Burnside rings or representations ring of groups.

John Greenlees

John has pioneered several developments in this field. For finite groups, all stable rational questions reduce to the well understood algebraic theory of Mackey functors. For compact Lie groups of positive dimension, the story is much richer, and John initiated the algebraization program for rational equivariant stable homotopy theory by proposing a compelling algebraic model. The model is suspected to be a faithful image of the geometry, its homological complexity is bounded by the rank of the Lie group, and it is suitable for "hands on" calculations. John verified the correctness of the model in several individual cases and, in joint work with Shipley, for all tori. John's conjecture that the model correctly describes rational stable equivariant homotopy theory for every compact Lie group is one of the open challenges in the field.

In another direction, John's work provides deep insight into the nature of equivariant complex orientability, leading to the discovery, with Cole, of equivariant formal group laws and, with May, to far reaching localization and completion theorems for theories on which equivariant bordism acts. Here multiplicative norm maps, originally introduced in group cohomology by Evans, are exploited for calculations in homotopy theory for the first time. Such norm maps exist in bordism, K-theory and various other geometrically flavored equivariant theories, and their study is an active area of current research. John's invention of the "correct" equivariant form of connective K-theory belongs here as well, which was motivated by the desire for complex orientability and a completion theorem. While the precise nature of homotopical equivariant bordism is still somewhat mysterious, the things we do understand are to a large extent due to John's work.

John has been intrigued by duality properties, both in commutative algebra and in "algebra" over structured ring spectra, another central theme of current stable homotopy theory. A highlight of his work in this direction is the paper with Dwyer and Iyengar about "Duality in algebra and topology." The title says it all: using structured ring spectra, a unifying framework for various seemingly unrelated duality phenomena is created, including Poincaré duality for manifolds, Gorenstein duality for commutative rings, or Gross–Hopkins duality in chromatic stable homotopy theory. John's beautiful theoretical insights often lead to effective methods of calculation. A good example is the use of local cohomology for calculating equivariant homology of universal spaces of groups, eventually leading him to discover homotopical counterparts of Gorenstein properties for ring spectra.

— Stefan Schwede

Expanding Math Circles with Seed Grants

Outreach Highlights from 2013–14

Peter Trapa

"This isn't like the math we do in school!" is one of the common refrains you frequently hear at a Math Circle — and that is precisely the point.

Mathematicians know that the practice of mathematics is fun, exciting, tremendously rewarding, often frustrating, and more than a little bit addictive. But most mathematicians developed this appreciation only in graduate school. How do we reach students earlier?

Math Circles continue to provide one kind of answer. Math Circles leaders guide students through iterations of experimentation and conjecture while developing new techniques to tackle problems that may take hours or weeks (or even months) to understand. In other words, leaders guide students in the practice of mathematics mentioned above. Anyone who has attended a Math Circle can attest that students quickly become hooked.

The potential for Math Circles is great: they can exist anywhere there are bright kids and a sophisticated leader. The National Association of Math Circles (NAMC), founded by MSRI in 2009, continues to tap into this potential. Twenty-seven seed grants were awarded over two funding cycles over the past year, and www.mathcircles.org continues to roll out new features to support the Math Circle community.

Puzzles Column

Elwyn Berlekamp and Joe P. Buhler

1. Place the 12 vertices of a regular icosahedron on the surface of the earth, with one vertex at the North Pole. *Estimate* which of the following latitudes lies closest to the other five vertices in the Northern hemisphere:

(a) the Arctic Circle, at 66.56° N;
(b) the US/Canada border, at 49° N;
(c) the Arizona/Mexican border, at 31.34° N;
(d) the Tropic of Cancer, at 23.44° N.

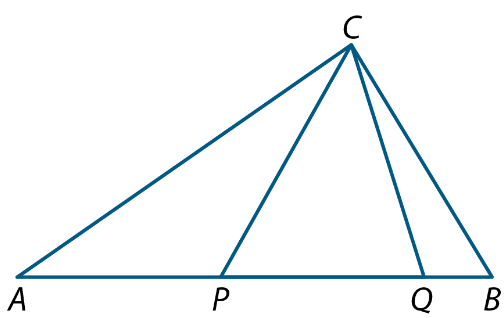
Now assume that the earth is a perfect sphere and calculate the exact latitude of those vertices.

2. Suppose that x and y are real numbers such that

$$\left(x+\sqrt{1+x^2}\right)\left(y+\sqrt{1+y^2}\right)=1.$$

Find $x+y$. (A hint is appended at the end.)

Comment: This problem, and the next one, were on the 2014 Bay Area Mathematical Olympiad. The next problem was originally due to our prolific problem composer Gregory Galperin.



3. Let ABC be a scalene triangle with the longest side AB. Let P and Q be the points on the side AB such that $AQ = AC$ and $BP = BC$. Show that the circumcenter of CPQ (the center of the circle through those three points) is equal to the incenter of ABC (the center of the circle that is tangent to the three sides of that triangle).

4. Divide the set of fractions $1/2, 2/3, 3/4, 4/5, \ldots, 99/100$ into two sets of sizes m and n (where $m+n=99$) in such a way that the two sets have the same product.

(a) What is the smallest possible value of m?
(b) What are all possible values of m?

5. Suppose that u, v, z, w are complex numbers each a distance 1 from the point 1 in the complex plane. Prove that if $uv = zw$ then $u = z$ or $u = w$. (A hint is appended at the end.)

6. Alice, Bob, and Charlie play the following game. Alice and Bob have a strategy session, after which they do not communicate except as implicit in the protocol below. Charlie picks a secret string of n bits. For each bit in turn the following things happens, in order:

1. Alice publicly guesses Charlie's bit.
2. Bob publicly guesses Charlie's bit, after hearing Alice's guess.
3. After both guesses, Charlie reveals the bit to both Alice and Bob.
4. Charlie gives \$1 to both players if and only if both guesses are correct.

Clearly in this game Alice and Bob have an expectation of $n/4$ dollars if they guess randomly and $n/2$ dollars if Bob echoes Alice.

However, one day Bob is able to steal Charlie's string beforehand. (Alice and Bob knew that this would happen prior to their strategy session, but Bob stole the string later, so Alice does not know the string while the game is played.) Here are three practical questions:

(a) If $n=5$, find a strategy that guarantees \$3 for Alice and Bob.
(b) If $n=8$, find a strategy that guarantees \$5 for Alice and Bob.
(c) If $n=9$, find a strategy that guarantees \$7 for Alice and Bob.

Finally, if $f(n)$ is the maximum possible guaranteed win for Alice and Bob using the best possible strategy for n-bit strings, show that $c = \lim f(n)/n$ exists and that c is the solution to the equation

$$3^{1-c} = 2c^c(1-c)^{1-c}.$$

Comment: This problem (actually, part (c)) was the September 2013 problem on the IBM problem site "Ponder This" run by Oded Margalit.

Hints. *For problem 2: $x+y=0$. For problem 5: This is taxing if done directly. A useful quote from Wikipedia: "Many difficult problems in geometry become much more tractable when an inversion is applied." That is, try to characterize the reciprocals of all points on the circle in the problem.*

Clay Senior Scholarships

The Clay Mathematics Institute (www.claymath.org) has announced the 2014–2015 recipients of its Senior Scholar awards. The awards provide support for established mathematicians to play a leading role in a topical program at an institute or university away from their home institution.

Here are the Clay Senior Scholars who will work at MSRI in 2014–2015:

Geometric Representation Theory (Fall 2014)
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New Geometric Methods in Number Theory and Automorphic Forms (Fall 2014)
Pierre Colmez, Institut de Mathématiques de Jussieu

Dynamics on Moduli Spaces of Geometric Structures (Spring 2015)
Marc Burger, ETH Zürich

Geometric and Arithmetic Aspects of Homogeneous Dynamics (Spring 2015)
Elon Lindenstrauss, Hebrew University

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