Geometric Models for Noncommutative Algebras

— Errata —

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page 7, Exercise 4
Pol Vanhaecke has pointed out that the answer to questions 1 and 2 is frequently "no." Therefore we rephrase the problem as: "Is the Lie ideal generated by the image of J equal to the kernel of i?"

page 50, line 6
Corollary 8.4 is a trivial fact, which does not depend on the hard theorem of von Neumann. In fact, it follows from the following properties of the commutant operation, $\mathcal{A} \mapsto \mathcal{A}'$:

1. $\mathcal{A}$ is contained in $\mathcal{A}''$, and
2. if $\mathcal{A}$ is contained in $\mathcal{B}$, then $\mathcal{B}'$ is contained in $\mathcal{A}'$.

page 52, line 4
This question has a trivial affirmative answer and should be deleted. See the correction to page 50, line 6.

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page 54, Proposition 9.2
Remove "complete" from the first sentence.

pages 54 and 55, proof of Proposition 9.2
Replace the full text of the proof by the following text:

"Let $F_j \subseteq TM$ be the distribution spanned by the hamiltonian vector fields of functions in $J_j^*(C^\infty(P_j))$. The assumption says that, at each point, the distribution F_1 (respectively F_2) gives the subspace tangent to the fibers of J_2 (respectively J_1); this shows that each of F_1 and F_2 is integrable. The (singular) distribution $F_1 + F_2$ is integrable as well. In fact, for each symplectic leaf $\mathcal{O}_1$ of P_1, the set $J_1^{-1}(\mathcal{O}_1)$ is a connected integral manifold of $F_1 + F_2$, and these inverse images fill M. But they are also the inverse images $J_2^{-1}(\mathcal{O}_2)$ of leaves in P_2. Hence, we get a map $\mathcal{L} \mapsto (J_1(\mathcal{L}), J_2(\mathcal{L}))$ from the leaf space of $F_1 + F_2$ to the product of the leaf spaces of P_1 and P_2. The image $\mathcal{R}$ of this map gives a relation between the leaf space of P_1 and the leaf space of P_2. Additionally, the projection of $\mathcal{R}$ to either factor of the product is bijective, because the fibers of J_1, J_2 are connected. It follows that $\mathcal{R}$ is the graph of a bijection."

page 55, line 15
Remove "and has constant rank".

page 55, line 19
Add the following new remark:
"**Remark.** The completeness of the J_i's and simply-connectivity of their fibers are used to relate Morita equivalence of Poisson manifolds to representation equivalence (see below). As we saw already in Proposition 9.2, some consequences of Morita equivalence do not require these assumptions."

page 73, line 1
The second "G" is redundant.

page 76, lines 4-6 from the bottom
These three lines, defining left and right translation maps, should read:

$$\text{“}\delta_g * \cdot : \begin{array}{ccc} \mathcal{D}'(G) & \longrightarrow & \mathcal{D}'(G) \\ \varphi & \longmapsto & \delta_g * \varphi \end{array}$$

and

$$\cdot * \delta_g : \begin{array}{ccc} \mathcal{D}'(G) & \longrightarrow & \mathcal{D}'(G) \\ \varphi & \longmapsto & \varphi * \delta_g \ . \end{array}$$

More concretely, for any test function $f \in C^\infty(G)$, we have

$$\langle \delta_g * \varphi, f \rangle = \langle \varphi, F \rangle \ , \qquad \text{where } F(x) = f(gx)$$

and

$$\langle \varphi * \delta_g, f \rangle = \langle \varphi, H \rangle \ , \qquad \text{where } H(x) = f(xg) \ .\text{”}$$

page 87, line 8
The diagram is missing the target space. The correct diagram is:

$$\begin{array}{c} X \times X \\ {\scriptstyle \pi_1} \downarrow\downarrow {\scriptstyle \pi_2} \\ X \end{array}$$

page 90, line 12 from the bottom
Replace "invariant" by "equivariant".

page 91, line 6
This line should read: "For $D \in \mathfrak{B}(\Gamma)$, let $\alpha(D) = Dg^{-1}$ and $\beta(D) = g^{-1}D$ for some $g \in D$. From..."

page 91, line 14
Replace "$g_2 D_1$" by "$D_1 g_2$".

page 93, line 13
Remove "and closed in G" and add "(for a more complete discussion of topological conditions on groupoids, see [143] or the book by Alan Paterson, Groupoids, Inverse Semigroups, and their Operator Algebras, Birkhäuser, 1999)".

page 93, Exercise 4
Part (c) should read: "the map $\alpha \times \beta : G \times G \to G^{(0)} \times G^{(0)}$ is transverse to the diagonal."

page 115, Historical Remark
Rewrite the paragraphs on this page as follows:
"**Historical Remark.** Already in 1963, Rinehart [145] noted that, if a Lie algebra Γ over a field k is a module over a commutative k-algebra C, and if there is a homomorphism ρ from Γ into the derivations of C, then the semidirect product Lie bracket on the sum $\Gamma \oplus C$ defined by the formula

$$[(v,g),(w,h)] = ([v,w], \rho(v)\cdot h - \rho(w)\cdot g)$$

satisfies the Leibniz identity

$$[(v,g), f(w,h)] = f[(v,g),(w,h)] + (\rho(v)\cdot f)(w,h) \quad \text{for } f \in C$$

if and only if the bracket on Γ satisfies the identity

$$[v, fw] = f[v,w] + (\rho(v)\cdot f)w \quad \text{for } f \in C\ .$$

In the special case where $C = C^\infty(X)$, the $C^\infty(X)$-module Γ, if projective, is the space of sections of some vector bundle E over X. The homomorphism ρ and the Leibniz identity imply that ρ is induced by a bundle map $\rho : E \to TX$.

In 1967, Pradines [139] coined the term "Lie algebroid" and proved that every Lie algebroid comes from a (local) Lie groupoid.[1] He asserted that the local condition was not needed, but this was later shown by Almeida and Molino [9] to be false. (See Section 16.4.)

Rinehart [145] proved (in a more algebraic setting) an analogue of the Poincaré-Birkhoff-Witt theorem for Lie algebroids. He showed that there is a linear isomorphism between the graded version of a universal object for certain actions of the pair $(\Gamma(E), C^\infty(X))$ on sections of vector bundles V over X, and the polynomials on the dual of the Lie algebroid E. As a result, the dual bundle of a Lie algebroid carries a Poisson structure. This Poisson structure is described abstractly in [34] as the base of the cotangent groupoid T^*G of a Lie groupoid G; it is described more explicitly in [35]. (See Section 16.5.) For more on "Lie-Rinehart algebras" and their universal objects, see [83]."

page 119, line 8
Replace "The Leibniz identity and Jacobi identity" by "The Jacobi identity (together with the Leibniz identity) and the fact that $\Gamma(\rho)$ is a Lie algebra homomorphism".

page 119, lines 10 and 11
The indices for the ξ's should be $1, \ldots, r$.

page 123, lines 13 and 14
The first sentence should read: "Bisections of this groupoid correspond to "gauge transformations covering diffeomorphisms", that is, automorphisms (*i.e.* H-equivariant diffeomorphisms) of the principal bundle."

page 131, line 16
The differential operator $d_{_E}$ is $\mathbb{R}$-multilinear, and not $C^\infty(X)$-multilinear.

page 144, lines 18 and 19
Replace "can be deformed to a morphism... cohomology" by "is a so-called L^∞ isomorphism of differential graded Lie algebras".

page 150, last line
Insert ", as a vector space," after "symmetrization".

[1] Robert Bryant has pointed out that the fact that every real-analytic Lie algebroid is locally integrable to a local groupoid is essentially contained in work of E. Cartan (see the appendix to R. Bryant, Bochner-Kähler metrics, preprint `math.DG/0003099`).