Math 53 Final, Summer 2009

Friday, Aug 14 8:10-10:00 AM Instructor: Shuchao Bi

Name:

Instructions: Each question is worth 10 points. Questions are written on both sides of the paper. To get full credit for a question, you must obtain the correct answer, box the correct answer, and show correct work (to avoid losing points, cross out incorrect work). No calculators or notes permitted. Good luck!

1. Find the absolute maximum and minimum values of $f(x, y) = x^2 + y^2 - xy - 3x$ on the region $x^2 + y^2 - xy \leq 9$. Also state the points at which these values occur.

2. Find the volume of the solid formed by the intersection of $x^2 + y^2 = 1$ and $y^2 + z^2 = 1$.

3. Evaluate $\int_0^2 \int_0^{\sqrt{9-(9/4)x^2}} e^{(9x^2+4y^2)} dxdy$.

4. Calculate $\int\int_R (x+y)e^{x^2-y^2}dA$ where R is the rectangle enclosed by lines $x-y=0$, $x-y=2$, $x+y=0$, and $x+y=3$.

5. Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the space curve $\mathbf{r}(t) = \langle t, sint, sint \rangle$, $0 \leq t \leq \pi$ and $\mathbf{F} = \langle x, sin(siny), cos(cosz) \rangle$.

6. Determine whether the vector field $\mathbf{F} = \langle 2xz + y^2, 2xy, x^2 + 3z^2 \rangle$ is conservative or not. If it is, find its potential function. If it is not, explain why.

7. Consider the vector field $\mathbf{F} = \langle \frac{-2y}{x^2+y^2}, \frac{2x}{x^2+y^2} \rangle$, find $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the ellipse $\frac{x^2}{4} + \frac{(y-1)^2}{9} = 1$ and oriented clockwise.

8. Find the area of the part of the surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 1$.

9. Evaluate $\int\int_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = xy\mathbf{i} + (y^2 + e^{xz^2})\mathbf{j} + sin(xy)\mathbf{k}$ and S is the surface of the region E bounded by the parabolic cylinder $z = 1 - x^2$ and the panes $z = 0$, $y = 0$, and $y + z = 2$.