

# UC Berkeley Math 10B, Spring 2015: Midterm 2

Prof. Sturmfels, April 9, 2015

**Name:** \_\_\_\_\_

**SID:** \_\_\_\_\_

**Neighbors:** Please write the names of the students next to you (or “None”):

Left: \_\_\_\_\_

Right: \_\_\_\_\_

**Section:** Circle your discussion section below:

**Grading**

Sec	Time	Room	GSI		
101	MWF 8-9	121 Latimer	Jason Ferguson	1ab	/ 5
102	MWF 9-10	104 Barrows	Jason Ferguson	2	/ 5
103	MWF 10-11	237 Cory	Minseon Shin	3	/ 5
104	MWF 11-12	121 Latimer	Indraneel Kasmalkar	4	/ 5
105	MWF 12-1	237 Cory	Indraneel Kasmalkar	5	/ 5
106	MWF 1-2	206 Wheeler	Joe Kileel	6ab	/ 5
107	MWF 2-3	283 Dwinelle	Joe Kileel	7	/ 5
108	MWF 9-10	110 Wheeler	Minseon Shin	8	/ 5
109	MWF 1-2	250 Dwinelle	Elina Robeva		
110	MWF 2-3	228 Dwinelle	Elina Robeva		

Other/none, explain: \_\_\_\_\_

/40

## Instructions:

- Closed book: No notes, no books, no calculators.
- The exam time is 80 minutes. Do all 8 problems.
- You must justify your answers for full credit.
- Write your answers in the space below each problem.
- Answers in complete sentences are encouraged.
- If you need more space, use reverse side or scratch pages. Indicate clearly where to find your answers.

1. (5 points) You are a pollster for the 2016 presidential elections. You ask 1000 random people whether they would vote for the Republican candidate or the Democratic candidate. 560 people say that they will vote for the Democrat.

(a) Find a 95% confidence interval for the percentage of votes  $p$  that the Democrat will get. (You do not need to simplify your answer).

(b) Based on the information you have, if you wanted to be 99% confident instead of 95%, will you have to increase the size of your confidence interval from part (a) or decrease it? Explain briefly in your own words.

2. (5 points) Find an explicit formula for the sequence  $\{a_n\}_{n \geq 0}$  which satisfies  $a_0 = 1$ ,  $a_1 = 4$ ,  $a_2 = 4$ , and

$$a_n = 2a_{n-1} + a_{n-2} - 2a_{n-3} \quad \text{for } n \geq 3.$$

**3.** (5 points) Is the function

$$f(x) = \sin x$$

a solution to the differential equation:

$$f'''(x) = f''(x) + f(x) \quad \text{for all } -\infty < x < \infty?$$

Why or why not? Give an explanation in complete sentences.

4. (5 points) Is the sequence defined by

$$a_n = \frac{1}{2}n(n+1) \quad \text{for } n = 0, 1, 2, 3, \dots$$

a solution to the recurrence relation:

$$a_n = 2a_{n-1} - a_{n-2} + 1 \quad \text{for all } n = 2, 3, 4, \dots?$$

Why or why not? Give an explanation in complete sentences.

5. (5 points) Find the general solution to the ordinary differential equation

$$(t^3 - t)y' = (t + 2).$$

6. (5 points) This table records the **observed frequencies** of joint outcomes for a random variable  $X$  with values  $x_1, x_2$  and a random variable  $Y$  with values  $y_1, y_2$ .

	$x_1$	$x_2$
$y_1$	17	12
$y_2$	11	8

- (a) Compute the table of **expected frequencies** under the null hypothesis  $H_0$  that  $X$  and  $Y$  are independent. (You do not need to simplify the numbers in your answer).

- (b) Explain how to use the tables of observed and expected frequencies to carry out a  $\chi^2$  test. Under what circumstances will we reject the null hypothesis?

7. (5 points) Determine all functions  $y(t)$  that satisfy

$$y'' + 4y' + 5y = 0 \quad \text{and} \quad y(0) = 3 \quad \text{and} \quad y(\pi/2) = 0$$

(Your final answer should not involve the complex number  $i = \sqrt{-1}$ .)

8. (5 points) Find all solutions to the differential equation

$$y' + y^2 \cdot \cos(t) = 0.$$

(Scratch Paper, Page 1)

(Scratch Paper, Page 2)