

March 5, 2004

Math 118

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First Midterm Exam

State your answers clearly and fully (with whole sentences, please). Include all your work. (Total points = 40.)

- 9 1.a) Give a precise definition of what is meant by an orthonormal **basis** for an infinite-dimensional vector-space with inner product. Make clear how you use the inner product (beyond for “orthonormal”).
- 5 b) Give a specific example of an orthonormal basis for $L^2([0, 1])$.
- 5 2.a) Give the definition of the convolution of two functions in $L^1(\mathbb{R})$.
- 9 b) Show that if g and h are filter functions in $L^1(\mathbb{R})$, and if L_g and L_h are the corresponding filter operators, then $L_g L_h = L_{g*h}$.
- 12 3. Let f be the function in $L^2([0, 1])$ defined on $[0, 1]$ by $f(t) = t^5$. Explain precisely what will happen for convergence of the Fourier series for f for
- mean-square convergence,
 - uniform convergence, and
 - pointwise convergence.
- Give reasons for your answers.