

MATH 1B, Lecture 3
Sarason

May 10, 1996

FINAL EXAMINATION

SIGNATURE _____

PRINTED NAME _____

SID Number _____

TA _____

Section time _____

Closed book except for crib sheet.
No calculators.

SHOW YOUR WORK. Cross out anything you
have written that you do not want the
grader to consider.

The points for each problem are in
parentheses. Perfect score = 145.

1. (10) Determine whether the improper
integral

$$\int_1^{\infty} \frac{d}{dx}(e^{-x} \ln x) dx$$

converges, and if it does, find its value.

1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
Total	
Grade points	

Name _____

2. (10) Perform the integration: $\int \frac{x^2+2x-1}{x(x^2+1)} dx$

Name _____

3. (10) Perform the integration: $\int \cos^3 x \sin^3 x \, dx$

Name _____

4. (15) Find the area of the surface of revolution one obtains by revolving about the x-axis the portion of the curve $y^2 = e^x + 1$ that lies between the lines $x = 0$ and $x = 1$.

Name _____

5. (15) Do the following infinite series converge? Explain your answers. In particular, make clear which convergence tests you are using.

(a) $\sum_{n=1}^{\infty} (n^{-2} + 2^{-n})$

(b) $\sum_{n=1}^{\infty} e^{-(n^{1/2})}$

Name _____

6. (15) Let k be a real number. (a) Find the n -th Taylor coefficient of the function $f(x) = (1+x)^k$ about the point $x = -\frac{1}{2}$ ($n = 0, 1, 2, \dots$). (b) Determine the radius of convergence of the Taylor series of f about the point $x = -\frac{1}{2}$ (same f as in part (a)).

Name _____

7. (10) Find the general solution of the differential equation

$$(x^2+4x+5)^{1/2}y' = y^2.$$

Name _____

8. (15) Let a vibrating system consist of a weight attached to a spring. Assume that, in suitable units, the mass of the weight is 1, the damping constant is 2, and the spring constant is 2. Assume the system is subjected to the driving force $F(t) = F_0 \sin t$ ($F_0 = \text{constant}$), and that it starts from rest in its equilibrium position at time $t = 0$. Find the formula giving the displacement as a function of time.

Name _____

9. (10) What is the 6th derivative of the function $f(x) = e^{-x^2}$ at the origin?

Name _____

10. (15) Find a numerical approximation to

$$\int_0^{1/10} \cos(x^2) dx$$

accurate to within 10^{-10} . Explain your method. (NO CALCULATORS)

Name _____

11. (20) (a) Use the method of power series to solve the initial value problem

$$(1-x^2)y'' - 6xy' - 4y = 0, \quad y(0) = 1, \quad y'(0) = 0.$$

(b) What is the radius of convergence of the series you obtain?

(c) Can you sum the series?