Your name:

Professor: Mariel Vázquez 23 March, 2001

Math128a: Numerical Analysis
Midterm Exam

Write solutions on these sheets.

Put your name on any extra sheet, and turn it in with your exam.

Justify all your answers.

You can use any result given in class, as soon as you state its contents correctly.

You do not need a calculator.

You start with 5 free points. **Good luck!**

Computations:

1. (5pts) Big $\mathcal{O}$ and little o notation.

Show that:

$$\frac{1}{n \ln n} = o\left(\frac{1}{n}\right)$$

2. (5pts)

What is x_2 if $x_0 = 1$, $x_1 = 2$, $f(x_0) = 2$, and $f(x_1) = 1.5$ in an application of the secant method. Illustrate your calculation with a figure.

3. (10pts)

Is the following function a natural cubic spline?

$$S(x) = \begin{cases} x^3 - 1, & \text{if } x \in [-1, 0]; \\ 3x^3 - 1, & \text{if } x \in [0, 1]. \end{cases}$$

Theory:

4. (10pts)

How would you use the loss of precision theorem to answer the following question: **How many bits of precision are lost in the substitution $1 - \cos x$ for a given x?** Remember to state the theorem.

5. (10 pts)

Mark each of the following true or false. (Justify your answers; if the statement is false give a counter-example.)

a. Is a function necessarily differentiable at a point where it is continuous? Is the converse true?

b. Does the function $f(x) = |x|$ possess Taylor expansion at $x = 0$?

6. (15 pts)

Let $p(x)$ be a polynomial of degree at most n that interpolates a given function f at $n+1$ prescribed nodes. Let L be a mapping that sends f to p.

a. Is L well-defined? Justify your answer.

b. Prove that L has the property that $L(q) = q$ for every polynomial q of degree at most n. Hint: Consider the function p-q and analyze its zeros.

Problems:

7. (20 pts)

a. Take n=2 and [a,b]=[0,1] in the Newton-Cotes procedure. Derive a formula to approximate $\int_0^1 f(x)dx$.

b. Use your result and a suitable change of variable to derive Simpson's rule for an arbitrary interval $[a, b]$.

Solve only one of the following two problems: (20 pts)

8. Devise a Newton iteration formula for computing $\sqrt[3]{R}$ where $R > 0$. Perform a graphical analysis of your function f(x) to determine the starting values for which the iteration will converge.

9.Prove that the point computed in the bisection method is the point where the line through $(a, sgn(f(a)))$ and $(b, sgn(f(b)))$ intersects the x-axis. Note: $sgn(f(x)) \in \{-1, +1\}$.

Extra credit problem (10pts)

Prove that if r is a zero of multiplicity k of the function f, then quadratic convergence in Newton's iteration will be restored by making this modification:

$$x_{n+1} = x_n - kf(x_n)/f'(x_n)$$

In the course of using Newton's method how can a multiple zero be detected by examining the behavior of the points $(x_n, f(x_n))$?