M. Rieffel   Math 128A   November 19, 1998
                      Second Midterm Exam

SHOW YOUR WORK COMPLETELY AND NEATLY.   Total points - 60.

1. a) Use the simple midpoint rule for numerical integration to obtain an
12   explicit multi-step method for solving ODE's. Include an explanation
            of your strategy.

12   b) Find the difference equation obtained by applying your multi-step
            method to the ODE $y' = \lambda y$. Obtain the characteristic equation for
            the difference equation, and its roots.

12   c) Using your answer to part b), obtain for appropriate λ's explicit
            parasitic solutions of the difference equation which grow while the
            true solution of $y' = \lambda y$ goes to 0.

4   d) Define what is meant by a <u>strongly stable</u> multi-step method, and
            show that the method you obtained above is not strongly stable.

2. Suppose you have an algorithm for computing approximations, T_h , to a
   number L , depending on a step-size h. Suppose you have reason to
   believe that the error in the approximation has the special form

$$L - T_h = ch^p + h^q r(h) .$$

   where $q > p$, the constant c is independent of h, with $c \neq 0$, and r
   is a bounded function of h .

13   a) Derive the formula for using this information to accelerate the
            convergence. Include an explanation of your strategy.

7   b) Explain precisely why you expect the accelerated convergence to
            indeed be faster, i.e. find the form of the error for the accelerated
            method.