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Fall 1997, Math H1A
Second Midterm

3 November, 1997
1:10-2:00 PM

1. (40 points, 8 points apiece) Compute each of the following. A correct answer gives full credit whether or not you show your computations. An incorrect answer, given with computations that are correct except for a minor error, will get partial credit.

(a) $\frac{d}{dx}\ln(e^x+1)$.

(b) $f'(5)$, where f is the inverse function to $g(x) = 2^x+3^x$.

(c) The maximum and minimum values of the function $f(x) = x + x^{-2}$ on the interval $[\frac{1}{2}, 2]$, and the values of x at which these occur.

(d) $\lim_{x\to 0}(e^x-1)/\sin x$.

(e) The general antiderivative of $\sin(px+q)$, where p and q are constants with $p\neq 0$.

2. (30 points) Derive the formula $\frac{d}{dx}\sin^{-1}x = 1/\sqrt{(x^2+1)}$. You may assume results proved in the text before that formula.

3. (30 points) (a) (15 points) Suppose f is a continuous function on a closed interval $[a,b]$, which is differentiable on the open interval (a,b), and that $f'(x) > 0$ for all points of that interval. Show that f is an *increasing* function on $[a,b]$. (This is Stewart's "Test for monotonic functions". In your answer, you may use anything proved in Stewart *before* that test.)

(b) (10 points) Suppose again that f is a continuous function on a closed interval $[a,b]$, but now let us only assume that for some $c\in(a,b)$, f is differentiable and has positive derivatives on each of the open intervals (a,c) and (c,b) (but not necessarily at c). Show that in this situation also, f must be an increasing function on all of $[a,b]$.

In proving this you may use the result of (a) (even if you did not succeed in proving it!), but you may not use the "generalized criterion for increasing functions" that I proved in class (of which this is a special case).

(c) (5 points) Give an example of a function f satisfying the assumptions of (b), but not all the assumptions of (a), for real numbers a, b, c which you should specify. You are not asked to prove that f has the properties required.