Math H1A Fall, 1995

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Midterm Exam—November 2, 1995

This is an "open book" exam. You may consult your notes and textbook. Grading is based on completeness, clarity, and accuracy. Please write in complete English sentences and explain your reasoning carefully.

1 *(4 points).* Evaluate $\lim_{b \to 2} \dfrac{b^{691} - 2^{691}}{b - 2}$ by using rules of differentiation, first expressing the limit as a derivative.

2 *(6 points).* Let c be a real number. Use Rolle's Theorem to show that the equation $x^5 - 6x + c = 0$ has at most one solution in the interval $[-1, 1]$.

3 *(6 points).* Using techniques of differentiation, find the equation of the line tangent to $y = \sqrt[3]{x^2 - 1}$ at the point $(-3, 2)$.

4 *(6 points).* Use l'Hôpital's Rule to evaluate $\lim_{\theta \to 0} \dfrac{\cos\theta - 1}{\theta^2}$.

5 *(8 points).* Let S be a non-empty bounded set of real numbers. Suppose that neither $a := \inf S$ nor $b := \sup S$ is in S.

(i) For each $x \in (a, b)$, show that there are numbers t and u in S which satisfy $a < t < x$ and $x < u < b$.

(ii) Assume now that S has the following property: if S contains two numbers t and u, then S contains all numbers between t and u. Prove that $S = (a, b)$.