Math 113: Introduction to abstract algebra.

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Midterm, Wednesday October 18, 11–12 a.m., 2 Evans.

Name:

Note. You have to do **three** out of the four problems. Cross out the problem that you don't want to be graded. Theorems proved in the book or in class may be used without proof (but do give the formulation).

1	
2	
3	
4	
Total	

Problem 1.

Define $\sigma, \tau \in S_7$ by

$$\sigma = (1\ 2)(3\ 4)(5\ 6)$$
$$\tau = (2\ 3)(6\ 7)$$

Write $\sigma\tau$ as a product of disjoint cycles, and compute the order of $\sigma\tau$. Is $\sigma\tau$ even or odd? Explain your answers.

Solution:

Problem 2.

Let G be the group $\mathbb{Z} \times \mathbb{Z}$, and let $a = (6,9) \in G$. Prove that $G/\langle a \rangle$ has an element of order 3.

Solution:

Problem 3.

Denote by $\mathbb{R}$ the additive group of real numbers, and by $\mathbb{C}^*$ the multiplicative group of non-zero complex numbers. Let $f\colon \mathbb{R} \to \mathbb{C}^*$ be the homomorphism defined by

$$f(x) = \cos(2\pi x) + i\sin(2\pi x)$$

(you do not have to prove that f is a homomorphism).

Determine the image of f and the kernel of f. Explain your answer. Prove that

$$\mathbb{R}/\mathbb{Z} \cong \mathbb{T},$$

where $\mathbb{Z}$ is the group of integers and $\mathbb{T}$ is the circle group: $\mathbb{T} = \{z \in \mathbb{C} \mid |z| = 1\}$.

Solution:

Problem 4.

Let $G = \mathbb{Z}_3 \times \mathbb{Z}_{10}$, and let Q be the quaternion group. How many homomorphisms $G \to Q$ are there? Explain your answer.

Solution: