Math H53 Midterm Exam 2

November 5th, 2003

Dr. K. G. Hare

1 a: (5 pts) Use Lagrange multipliers to find the maximum of $f(x, y) = x^2 - y^2$ given the constraint $x^2 + 2y^2 = 1$.

2 a: (5 pts) Evaluate

$$\int_0^1 \int_0^{\sqrt{1-x^2}} e^{x^2+y^2}\, dx\, dy$$

by converting to polar coordinates

b: (5 pts) Let $f(x, y) = |x| + |y|$. Find the Fenchel subdifferiential $\partial_c(f(x, y))$.

3 a: (5 pts) Let $\mathcal{A}$ be the set of all possible stories written in English. Show that $\mathcal{A}$ is countable.
Bonus: (2 pts) Show the set of all possible *illustrated* stories is uncountable.

4 a: (5 pts) Let $\langle u(x, y), v(x, y)\rangle$ be a conservative vector field, with a potential function $U(x, y)$. Let $\langle v(x, y), u(x, y)\rangle$ be a conservative vector field, with a potential function $V(x, y)$. Show that $\langle U(x, y), V(x, y)\rangle$ and $\langle V(x, y), U(x, y)\rangle$ are both conservative vector fields.

b: (5 pts) Reverse the order of integration of

$$\int_0^1 \int_{x^2}^{x} f(x, y)\, dx\, dy.$$

(Do not evaluate.)

5: (5 pts) Let

$$f(x, y) = \frac{2}{3}(x-1)^{\frac{3}{2}} + \frac{2}{3}y^{\frac{3}{2}}.$$

Find the surface area of $f(x, y)$ over the region $D = \{(x, y) : 0 \le x \le 1, 0 \le y \le 1\}$.

6: (5 pts) Let $T : \mathbb{R} \to \mathbb{R}^2$ by $T(x, y) = \langle e^x \cos y, e^x \sin y\rangle$ Let $R = \{(x, y) : 1 \le x^2 + y^2 \le 4\}$. Us the change of variables T to evaluate

$$\iint_R \frac{1}{\sqrt{u^2 + v^2}}\, du\, dv$$