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Fall 2003, Math 104, Sec. 2

Second Midterm

27 Oct., 2003
11:10-12:00

1. (27 points, 9 points each.) Complete the following definitions. You may use, without defining them, terms or symbols which Rudin defined before he defined the word asked for. You do not have to use exactly the same words as Rudin, but for full credit your statements should be clear, and be logically equivalent to his.

(a) Suppose that f is a function from a subset E of a metric space X to a metric space Y, that p is a limit-point of E, and that q is a point of Y. Then we say $\lim_{x \to p} f(x) = q$ if ...

(b) A metric space X is said to be *complete* if ...

(c) Let f be a real-valued function on a segment $(a, b) \subseteq R$, and let $x \in (a, b)$. Then f is said to have a *simple discontinuity* at x if ...

2. (36 points; 9 points each.) For each of the items listed below, either *give an example* with the properties stated, or give a brief reason why *no such example exists.*

If you give an example, you do *not* have to prove that it has the property stated; however, your examples should be specific; i.e., even if there are many objects of a given sort, name a particular one. If you give a reason why no example exists, don't worry about giving a detailed proof; the key relevant fact will suffice.

(a) A covering $\{G_\alpha\}$ of the segment $(-1, 1)$ by infinitely many open sets $G_\alpha \subset R$, which has a finite subcovering.

(b) A convergent series $\Sigma\, a_n$ such that the series $\Sigma\, |a_n|$ is divergent.

(c) A power series $\Sigma\, a_n z^n$ with radius of convergence 17.

(d) A differentiable function f on an interval $[a, b]$ such that f' is not everywhere continuous on $[a, b]$.

3. (19 points.) Suppose f is a continuous real-valued function on an interval $[a, b]$ which is differentiable on the segment (a, b), and suppose there is a point $c \in (a, b)$ such that $f(c) > f(a)$ and $f(c) > f(b)$. Show that there is a point $d \in (a, b)$ such that $f'(d) = 0$.

4. (18 points.) If $I_1 \supset I_2 \supset \ldots \supset I_n \supset \ldots$ are intervals in R, show that $\bigcap_{n=1}^{\infty} I_n \neq \varnothing$.

(Yes – this is the problem that was dropped from the first midterm because of confusion on notation. I've made the notation explicit. Here's your chance to get it right.)