

Math 53 Midterm #1, 10/2/03, 8:10 AM – 9:30 AM Hutchings

No calculators or notes are permitted. Each of the 6 questions is worth 10 points. Please write your solution to each of the 6 questions on a separate sheet of paper with your name on it. Please put a box around the final answer and, to maximize credit, show your work. Good luck!

1. (a) Sketch the curve given in polar coordinates by the formula

$$r = e^{2\theta}, \quad 0 \leq \theta \leq 2\pi.$$

- (b) Compute the length of the above curve.

2. Show that the limit

$$\lim_{(x,y) \rightarrow (1,1)} \frac{x + 2y - 3}{x + y - 2}$$

does not exist.

3. Suppose the function $f(x, y)$ satisfies

$$\frac{\partial f}{\partial x} = \frac{1}{x}, \quad \frac{\partial f}{\partial y} = \frac{1}{y}.$$

Suppose as usual that $x = r \cos \theta$ and $y = r \sin \theta$. Calculate $\partial f / \partial r$ and $\partial f / \partial \theta$ in terms of r and θ .

4. (a) Sketch the surface $x^2 + (y - 1)^2 = z^2$.
 (b) Find the tangent plane to the above surface at the point $(4, 4, 5)$.
 (Write your answer in the form $ax + by + cz = d$.)
5. (a) Find normal vectors to the planes

$$2x + 2y + z = 5, \quad 2x - y - 2z = -1.$$

- (b) The above two planes intersect along a line L . At what angle do the two planes intersect? *Hint:* this is the angle between the above two normal vectors.
 (c) Find a tangent vector to the above line L . *Hint:* this must be perpendicular to the above two normal vectors.
6. Suppose z is implicitly defined as a function of x and y by the formula

$$ze^{x+2y+3z} = 4.$$

Calculate $\partial z / \partial x$ and $\partial z / \partial y$ in terms of x, y , and/or z .