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Fall 2003, Math 104, Sec. 2

First Midterm

22 Sept., 2003
11:10-12:00

1. (27 points, 9 points each.) Complete the following definitions. You may use, without defining them, terms or symbols which Rudin defined before he defined the word asked for. Your definitions do not have to use exactly the same words as Rudin's, but for full credit they should be clear, and be logically equivalent to his.

(a) An ordered set S is said to have the *least upper bound property* if ...

(b) A subset E of a metric space X is said to be *open* if ...

(c) A subset α of the field Q of rational numbers is called a *cut* if ...

2. (36 points; 9 points each.) For each of the items listed below, either *give an example,* or give a brief reason why *no example exists.*

If you give an example, you do *not* have to prove that it has the property stated; however, your examples should be specific; i.e., even if there are many objects of a given sort, name a particular one. If you give a reason why no example exists, don't worry about giving reasons for your reasons; a simple statement will suffice.

(a) A positive real number c such that $\{nc : n \in J\}$ is bounded above (where J denotes the set of positive integers).

(b) A countable subset $E \subset R$ and a point $x \in R$ such that x is a limit point of E.

(c) Two vectors $\mathbf{x}, \mathbf{y} \in R^k$ (for some integer k) such that $\mathbf{x} \cdot \mathbf{x} = 9$, $\mathbf{x} \cdot \mathbf{y} = 15$, $\mathbf{y} \cdot \mathbf{y} = 16$.

(d) A family of open sets $V_n \subset R$ (where n ranges over all the positive integers) such that $\bigcap_n V_n$ is not open.

3. (19 points.) Let p be a point of a metric space X and r a positive real number, and let $M = \{q \in X : d(p, q) > r\}$. Prove that M is open in X.

4. (18 points.) If $I_1 \supset I_2 \supset \ldots \supset I_n \supset \ldots$ are intervals in R, show that $\bigcap_n I_n \neq \varnothing$. (This was a result proved in the reading. For full credit, you should explain more precisely than Rudin does why the element you obtain belongs to all the I_n.)