Math 191: High School Math from an Advanced Standpoint

Fall 2002
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Midterm Exam

Instructions: This exam consists of four problems. Please write your solutions clearly, showing your work and explaining your reasoning. Please label, or define, any variables and notation you choose to work with. Keep this sheet for yourself if you wish.

   You may work with other people in the class. If you do, you should still write your own solutions and, once you start writing, you should work alone. If you work with others, please write their names. You should not work with anyone from outside this class.

   Each problem is worth 25 points, for a total of 100 points.

1.(a) Prove that $\log_2 5$ is irrational.

(b) Generalize part (a) and prove your generalization.

2. Show that $\sin \frac{2\pi}{5} = \sqrt{\frac{5+\sqrt{5}}{8}}$.

3. If two triangles have equal area and equal perimeter, must they be congruent? Prove your answer.

4. This is a "problem analysis" in the spirit of the average and of the water evaporation problems of chapter 1. We start with a problem typical of high school texts.

(a) Find the $x-$ and $y-$ intercepts of the line through the point $(5,2)$ that cuts off the triangle of smallest area in the first quadrant.

(b) What geometric property relates the point $(5,2)$ to the solution you found in part **a**? Generalize your result by replacing $(5,2)$ with an arbitrary point (p,q) in the first quadrant. Check whether or not (p,q) always has the same geometric property in relation to the line segment that minimizes the area.

(c) If you used calculus to solve part (b), can you come up with a solution which is purely geometrical and uses no calculus? If you did not use calculus in part (b), can you get a solution using calculus?

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