Final Exam, Math 16B, Summer 2009

Name: ____

Student ID #: ____

By turning in this exam, completed, you are indicating that you did your own work and are in compliance with all university academic honesty policies. Evidence of less than original work will result in a grade of zero.

There are 7 pages and 10 questions on this exam, in addition to the take home portion of the exam. By turning it in, you are indicating that you are not missing pages, problems, or access to any part of the test that would unfairly lower your score by no fault of your own.

For instructor:

(1) _____
(2) _____
(3) _____
(4) _____
(5) _____
(6) _____
(7) _____
(8) _____
(9) _____

(10) _____

home _____

Exam Score __________________________ / 120

General Comments:

(   ) (12 points) Read carefully ----

The famous "Lotka-Volterra" equations for modeling predator/prey relationships are given by:

$$\frac{dx}{dt} = x(\alpha - \beta y) = \alpha x - \beta xy$$
$$\frac{dy}{dt} = -y(\gamma - \delta x) = \delta xy - \gamma y$$

where α, β, γ, δ are constants, **x represents the size of population of the prey**, **y represents the size of the population of predators,** and t is time [unit-less].

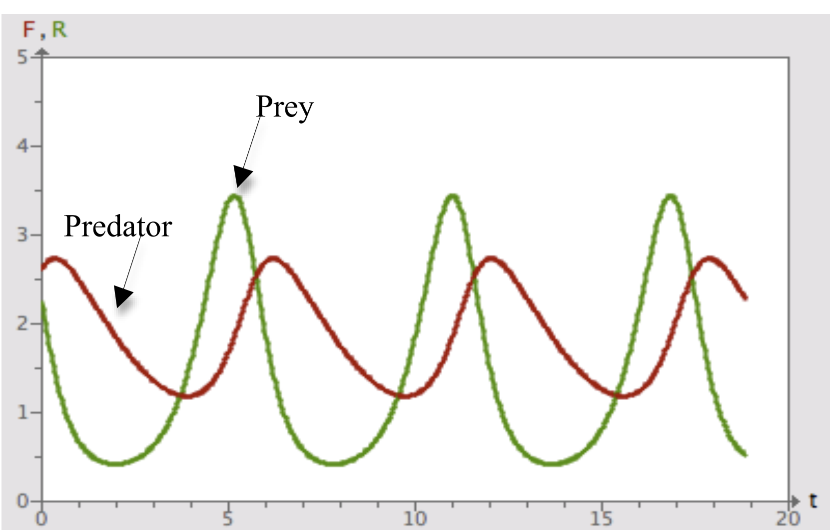
(a) Below are four English descriptions of the four parameters α, β, δ, γ. Please match the parameter with its description.

______Population growth constant for predators   ______Predation rate [predators on prey]

______Population growth constant for prey   ______ Rate of natural death of predators

(b) What are some real-life parameters that impact predator/prey relations that are not captured by this model? [in other words, what assumptions do you make by using this model]

(c) Below is a graph showing the size of predator and prey populations over time. Explain what's happening on the graphs by pointing out interesting features and then commenting on them. Be sure to give the 'full picture' by explaining at least one full cycle of what's happening.



(   ) (8 points) Find the following for $f(x, y) = \sin(xy) + 2xye^{y}$

(a) $\dfrac{\partial^2 f}{\partial x^2}$ ____

(b) $\dfrac{\partial f}{\partial y}$ ____

(   ) (8 points) If $a_k > 0$ <u>and</u> $\sum_{k=1}^{\infty} a_k$ converges, tell me why $\sum_{k=1}^{\infty} a_k^2$ must also converge.

(   ) (8 points) Construct the following:

(a) A geometric series that diverges
(b) A geometric series that converges
(c) A geometric series that sums to something smaller than 1
(d) A geometric series with an alternating term

Solutions:

(a) ____
(b) ____
(c) ____
(d) ____

(   ) (12 points) Consider the function $f(x,y) = (x^2 + y^2)e^{xy}$. Which of the following statement(s) are **true** about *f(x,y)*? [show some work or say something that could back up your conclusion, or you will not receive full points --- i.e. guessing will not help you]

(a) *f(x,y)* has a critical point at *(x,y)=(0,0)*

(b) The level curve of *z=f(x,y)* which contains the point *(x,y)=(1,0)* occurs when *z=1.*

(c) *f(x,y)* has a local maximum

(   ) (8 points) Find the general solution to: $\cos^3(t)y' = 3\cos^2(t)\sin(t)y + t\ln(t)$

(   ) (10 points) Suppose that the amount of time (in minutes) a person spends waiting in line at a movie theater is a random variable with the density function $f(x) = \dfrac{1}{72}x$. We know that we might get lucky and not have to wait in line at all, so the lower bound of the pdf is 0 [minutes]. What is the upper bound of the pdf? What is the <u>average time</u> spent waiting to see your movie.

[hint – the numbers work out very nicely in the end]

Upper bound of the pdf is: ____

Average time spent is: ____

(   ) (10 points) from your textbook, and also very similar to the one on the second midterm…☺

Morphine is a drug that is widely used for pain management. However, morphine can be fatal by causing respiratory arrest. Since pain perception and drug tolerance vary with patients, morphine is gradually administered with small increments until pain is controlled or side effects appear.

In one intravenous infusion, morphine was injected continuously at an increasing rate of t milligrams per hour. Suppose that the body removes the drug at a rate proportional to the amount of the drug present in the body, with constant of proportionality 0.35. let $f(t)$ denote the amount of morphine in the body, t hours from the beginning of the infusion.

(a) Find a differential equation satisfied by $f(t)$
(b) Solve the differential equation from (a)

(   ) (9 points) Suppose you are playing a game of dice with your friend. You have two fair six-sided dice, labeled with the integers between 1 and 6 [just standard dice, guys].

(a) What is the probability that, when you roll your two dice, the sum of the two dice will be exactly 4?

(b) When you roll the dice, what is the most likely sum to occur? With what probability will that sum occur?

(   ) (15 points) Converge or diverge? Please **choose 5** of the following 6 problems and tell me:

-Whether they converge or diverge, making the strongest statement possible. Go as far as to say that the problem "converges to ___" if it is possible]

-What convergence or divergence *means* for the particular problem [put it in context – series, sequence, improper integral, etc…]

(a) $\sum_{n\geq 1} \frac{6}{5^n}$

(b) $\sum_{n\geq 0} \frac{4}{(n-2)^{1/2}}$

(c) $\int_0^\infty xe^{-x^2}\,dx$

(d) $\left\{\frac{n^2-n+2}{9-n^2}\right\}_{n\geq 1}$

(e) $\sum_{n\geq 3} e^{-n}$

(f) $\int_1^\infty \frac{e^{-x}}{1+e^{-x}}\,dx$

Take home portion of Final Exam, Math 16B summer 2009

These questions require some thought, and I don't want you to answer them while under time constraints during the final exam. Please complete the following questions and <u>bring it with you</u> to the final exam. It will constitute **<u>20 points of your final exam</u>**. I trust you to not discuss these with your fellow students, or use your text/notes to help you in answering them. Please include any work that you did to help you answer the questions [if you wrote anything down, that is].

(A)

> For continuous random variable x, *f(x)* is a pdf on *(a,d),* and $a \le b \le c \le d$.
> Are the following statements <u>True</u> or <u>False</u>?
> If false, make a (small) change that turns the statements into something true.

(1) $F(d) - F(a) = \Pr(x \le d)$

(2) $F(a) < F(c)$

(3) $F(a) < F(d)$

(4) $If\ \ c\ \ is\ \ the\ \ median\ \ value,\ \ then\ \ 1 - F(b) \ge \Pr(x \le c)$

(5) $F(c) - F(a) \le \Pr(x < b)$

(6) $F(c) = \Pr(x > c)$

(7) $\Pr(a \le x \le b) + (1 - F(c)) = 1$

(8) $\Pr(x = d) < \Pr(x = b)$

(9) $1 - F(b) = F(d) - F(b)$

(10) $\int_a^c f(x)dx \le \int_b^c f(x)dx$

(B) Two people are playing a dice game in which they roll 2 dice per turn. If the sum of the two dice is less than or equal to 6, person A gets a point. Otherwise, person B gets a point. The winner is the first to 10 points. Is this a fair game? Why or why not? If not, can you adjust something to make the odds equal [i.e. both players have equal chance of winning?]

(C) Two people are playing a game in which they flip 2 coins per turn. If the faces are the same (HH or TT), person A gets one point. Otherwise, person B gets a point. Winner is first to 10 points. If the score is tied, 9-9, who is more likely to win the game? Why?