GEBA

MATH 110 - MIDTERM 3/4 /2004

1. In $M_{m\times n}(F)$ define $W_1 = \{A \in M_{m\times n}(F) | A_{ij} = 0 \text{ whenever } i > j\}$ and $W_2 = \{A \in M_{m\times n}(F) | A_{ij} = 0 \text{ whenever } i \leq j\}$.

Prove that $M_{m\times n}(F) = W_1 \oplus W_2$.

2. Let u, v and w be distinct vectors of a vector space V. Prove that if $\{u, v, w\}$ is a basis for V, then $\{u + v + w, v + w, w\}$ is also a basis for V.

3. Let $T : M_{2\times 2}(\mathbb{R}) \to P_{\leq 2}(\mathbb{R})$ be the linear transformation defined by

$$T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = (a + b) + 2cx + dx^2$$

a) Find $N(T)$, $R(T)$, nullity(T), rank(T) by specifying a basis for each one of them.

b) Let α and β the standard bases for $M_{2\times 2}(\mathbb{R})$ and $P_{\leq 2}(\mathbb{R})$ respectively. Consider also $\gamma = \{1, x + 1, x^2 + 1\}$ basis for $P_{\leq 2}(\mathbb{R})$.

Compute Q the change of coordinate matrix from β-coordinates to γ-coordinates and the representation matrices $[T]_\alpha^\beta$,$[T]_\alpha^\gamma$.

Check that

$$[T]_\alpha^\gamma = Q \cdot [T]_\alpha^\beta.$$

4. Let V and W be finite-dimensional vector spaces and $T : V \to W$ be linear.

a) Prove that if dim $V <$ dim W, then T cannot be onto.

b) Prove that if dim $V >$ dim W, then T cannot be one-to-one.

5. Let A, B two square matrices, $A, B \in M_{n\times n}(\mathbb{R})$ such that $AB = A + B$. Prove that $AB = BA$.