Department of Mathematics
University of California, Berkeley

Quantitative Reasoning Examination - Solutions

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1

$$x^2 \left(\frac{15x}{7y^2}\right)\left(\frac{49y}{3x^3}\right) = \frac{(15)(49)(x^2)(x)(y)}{(7)(3)(y^2)(x^3)} =$$

$$(5)(7)\frac{x^3y}{x^3y^2} = \frac{35}{y}$$

2

If $a = -5$, then $|a+1| + |a-1| = |-5+1| + |-5-1| = |-4| + |-6| = 4 + 6 = 10$

3

$f(x) = x^2 - 4$, so $f(3) = 3^2 - 4 = 9 - 4 = 5$, and $f(f(3)) = f(3)^2 - 4 = 5^2 - 4 = 25 - 4 = 21$.

4

We are given

$$2x + 3y = 6,$$
$$3x + 2y = 14,$$

Multiplying the first equation by 3, the second by -2 and adding, we get $9y - 4y = 18 - 28$, giving $5y = -10$, and $y = -2$.

5

The polynomial $f(x) = x^2 - 2x - 15$ factors as $f(x) = (x+3)(x-5)$ from which one sees that $f < 0$ in the interval $(-3, 5)$, $f = 0$ at -3 and 5, and $f > 0$ on $(-\infty, -3)$ and $(5, \infty)$. Of the five choices in the question, only one, the interval $(-4, -3)$ lies in the region where $f > 0$.

6

An application of the Pythagorean theorem shows that the smaller square has side length $\sqrt{2}$, so that $S = 2 = \frac{1}{2}(4) = \frac{1}{2}L$, ie: $L = 2S$.

7

We use long division:

$$\begin{array}{r|rrr} & & x & -3 \\ \cline{2-4} x+2 & x^2 & -x & -5 \\ & x^2 & +2x & \\ \cline{2-3} & & -3x & -5 \\ & & -3x & -6 \\ \cline{3-4} & & & 1 \end{array}$$

The remainder is 1.

8

Recall that $log_y(x)$ is the power to which you must raise y to get x. Since $3^4 = 81$, $log_3(81) = 4$.

9

$$\frac{x+1}{x-1} + \frac{x-1}{x+1} = \frac{(x+1)^2 + (x-1)^2}{(x-1)(x+1)} = \frac{x^2+2x+1+x^2-2x+1}{x^2-1} = \frac{2x^2+2}{x^2-1}$$

10

$$\left(\frac{3ab^2}{c^3}\right)^{-4} = \left(\frac{c^3}{3ab^2}\right)^4 = \frac{c^{12}}{81a^4b^8}$$

11

30% of 15,000 equals (.30)(15,000) = (30)(150) = 4,500. The new DUH tuition will be 15,000 + 4,500 = 19,500 dollars per year.

12

By the quadratic formula, the roots of $x^2 - 8x - 1 = 0$ are

$$\frac{8 \pm \sqrt{64+4}}{2} = 4 \pm \sqrt{16+1} = 4 \pm \sqrt{17}.$$

Hence $4 - \sqrt{17}$ is a root.

13

The hypotenuse of the triangle has length $\sqrt{2^2+3^2} = \sqrt{13}$. The cosine of the angle in question equals the length of the adjacent leg divided by the length of the hypotenuse, hence equals $\frac{3}{\sqrt{13}}$.

14

If (x_θ, y_θ) is the point on the unit circle corresponding to an angle of θ radians, then the point $(-x_\theta, -y_\theta)$ corresponds to $\theta+\pi$ radians. We have $\cos(\theta) = x_\theta$, and $\cos(\theta+\pi) = -x_\theta$, giving $\cos(\theta+\pi) = -\cos(\theta)$.

15

We have $48 = log_2(x) + log_2(x^2) + log_2(x^3) = log_2(x) + 2log_2(x) + 3log_2(x) = 6log_2(x)$, so $log_2(x) = \frac{48}{6} = 8$, and $x = 3$

16

The function $\sin(\theta)$ increases from 0 to 1 on the interval $[0, \frac{\pi}{2}]$. Thus $\sin(\frac{\pi}{3}) > \sin(\frac{\pi}{4})$., i.e., (a) holds. (By the same token, (b) is false, and so is (d) because $\cos(\frac{\pi}{4}) = \sin(\frac{\pi}{4})$. The observation that $\cos(\frac{\pi}{2}) = 0$ eliminates (c) and the observation that $\cos(\frac{\pi}{6}) = \sin(\frac{\pi}{3})$ eliminates (e).)

Alternatively, if one remembers the sine and/or cosine of the angles appearing in this question, one can read off the answer: $\sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$, $\sin(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$, $\cos(\frac{\pi}{2}) = 0$, $\cos(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$, $\cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$.

17

$$\frac{\frac{x^2-y^2}{(x+y)^2}}{\frac{x+y}{(x-y)^2}} = \left(\frac{(x-y)(x+y)}{(x+y)^2}\right)\left(\frac{(x-y)^2}{x+y}\right) = \frac{(x-y)^3}{(x+y)^2}.$$

18

The line passes through $(0, 4)$ and has slope $\frac{4-0}{0-3} = -\frac{4}{3}$. Hence its equation is $y = 4 - \frac{4}{3}x$, which can be rewritten as $\frac{x}{3} + \frac{y}{4} = 1$.

19

The equation of the line is $y = 2 + 5(x - 1)$. For $x = 0$, we get $y = 2 - 5 = -3$. For $y = 0$, we get $x = -\frac{2}{5} + 1 = \frac{3}{5}$. The intersections with the coordinate axes occur at $(\frac{3}{5}, 0)$ and $(0, -3)$.

20

$$\frac{x^{7y+5}}{x^{5y-7}} = x^{7y+5-5y+7} = x^{2y+12}.$$