Riddles

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This is a compilation of our favorite brain-teasers. Some are written by us, but the majority are just ones we have heard from various sources. We claim no credit. Feel free to email us if you are stuck and want a hint, want to check your solution, or if you know of any good riddles we might want to add.

Note:

For each of the following riddles, all people are assumed to be perfect mathematicians with infinite computing power and memory.

Many riddles require mathematical intuition. Those which require a little bit of mathematical knowledge are marked with (+). Those which are silly given mathematical knowledge, but may be quite challenging without, are marked with (-). In all of the problems, the full power of ZFC is at your disposal.

Some of the settings involve a collection of mathematicians who have been imprisoned by a particularly whimsical and riddle loving villain. The villain sets a challenge for the mathematicians to solve in order to gain their freedom. The mathematicians are allowed to confer and come up with a strategy, but the villain will listen in on their plans. If they fail, the villain will keep them imprisoned forever.

Symbol Key:

(+) requires some basic (undergraduate-level) math

(-) More interesting for non-mathematicians

(n) (subjective) difficulty rating out of 5

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1 Too Many Hats

1.1 100 mathematicians and 2 hat colors (3)

100 mathematicians will be lined up all facing one direction and each will have a hat put on their head, either blue or red. Each mathematician will see all the hat colors of the people in front of them in line, but not their own hat color or the hat colors of anyone behind them. The warden will start at the back of the line and, one by one ask each mathematician their hat color. They are only allowed to press a blue or red button that announces either blue or red in a flat voice to the entire room.

The goal is for at most one mathematician to guess their hat color incorrectly.

1.2 (Countably) Infinitely many mathematicians (2)(+)

Now, there are countably infinitely many mathematicians in a line is of length ω (think $\mathbb{N}$) with the mathematicians facing towards the infinite direction.

The goal is for at most finitely many mathematicians to guess their hat color incorrectly.

1.3 ALL THE HATS! (3)(+)

Assume there are countably infinitely many mathematicians and each of them is wearing a hat labeled by one of countably many hat colors. The mathematicians are lined up as before, with each able to see all the hat colors of mathematicians further along. Come up with a strategy where at most one of the mathematicians gives an incorrect response.

2 More hats (5)

There are n mathematicians and n hat colors that they are wearing, possibly with repetition. All the mathematicians can see everybody else's hat color, but not their own. The warden will come up and ask them all to announce the color of their hat at the same exact time. Come up with a strategy for the mathematicians to follow so that at least one correctly guesses their own hat color.

3 Nickles on a table (4)(+)

You have 10 dots on a table. Can you always cover all of the dots with (non-overlapping) nickles?

Hint: a calculation you might end up using $\frac{\pi\sqrt{3}}{6} \sim .907$.

4 Ants on a log

4.1 Falling off (2)

You have n ants on a very thin 1m log. Each ant is either moving either left or right at 1m/s. When two ants bump into each other they immediately switch directions and begin moving in the opposite direction at the same speed. What is the longest it can take for all the ants to fall off the log?

4.2 Teleporting (3)

Now, when an ant reaches the end of the log, it teleports to the other end. How long does it take for you to guarantee that each of the ants have simultaneously returned to its original place, going its original direction?

5 Lightswitches

5.1 2 switches (3)

There are n mathematicians that will be locked into individual rooms in a facility and put into comas where they don't experience the passage of time. Every day, the warden will choose one of the mathematicians at random to wake up and bring to a room with two lightswitches. The mathematician is then forced to flip one of the two switches and brought back to their room and returned to a coma. The facility is then cleaned and all attempts at communication are eliminated with the exception of the state of the flipped lightswitches. Come up with a method by which one of the mathematicians will be able to say with absolute certainty that every other mathematician has been woken up at least once.

Hint: What if there is only one lightswitch and each mathematician is allowed to choose whether or not to flip the lightswitch?

5.2 A 3 colored switch (4)

Replace the two lightswitches with a single lightswitch that switches between three states in the room. Every mathematician must change the state of this switch when they enter the room.

6 Cards

6.1 Standard Deck (3)

You and a friend are preforming a card trick. You draw 5 cards at random from a shuffled deck of cards. You keep one card and give your friend the other four cards. Your friend now has to correctly guess the card you kept. What strategy can you two decide beforehand to ensure the friend guesses correctly?

6.2 Nonstandard Deck (5)(+):

How big of a deck is this possible with?

7 Number Machine

7.1 Two values (2)

You're given a machine to which you can input an integer and it outputs an integer. You know the machine just plugs your number into a polynomial and outputs the result, but you don't know which polynomial. You do know that the coefficients are all non-negative integers. The machine is at low battery, and only has energy left for two inputs. How can you guess the polynomial?

7.2 One value (1)(+)

Alternatively, you can switch the machine to real number mode, where you can input a real number. This takes twice the battery. Can you guess the polynomial this way?

8 Ropes (3)

You are given a large collection of ropes that each take exactly one hour to burn from one end to the other. Throughout this hour, each of these ropes burns at incredibly irregular rates (i.e. certain portions of the rope burn slowly while others burn very quickly) and each individual rope's rate is unrelated to any of the others.

Come up with a method by which to measure 45 min.

Hint: Can you measure 30 min?

9 Guessing Jelly beans (2)(-)

You are locked in a an empty room. Behind a curtain is a number of jars, each containing jelly beans. You don't know how many jars there are or how many jelly beans are in each jar. You can't see the jars. The door is unlocked when you correctly list out the number of jelly beans in each jar. (A correct guess looks like: "There are three beans in the first jar, five in the second, the third is empty. There are no more jars.") Can you come up with a method to start guessing so that you eventually get out of the room?

10 Names in Boxes (4)

There are 100 mathematicians and a room with 100 boxes that each contain the name of one of the mathematicians. Each mathematician will enter the room alone and wants to find the box with their name by opening at most 50 boxes. The mathematicians must come up with a strategy so that everyone opens the box containing their name. The one advantage they have is that the first person to enter the room is allowed to open all the boxes and switch exactly one pair of names.

11 Fortunately unfortunately (1)

Unfortunately, you come down with a deadly illness. Fortunately, there is a cure. Unfortunately the cure is very expensive. Fortunately, you are able to afford exactly the quantity of pills you need to recover: 100 of pill A and 100 of pill B. The pills all look identical. You need to take exactly one of each per day. Unfortunately, on day one, you put a pill A into your hand, but when you go to pour a pill B into your hand, two fall out into your hand, and now are mixed in with the pill A. If you take all three pills, you'll overdose on pill B. Fortunately, you have some time to devise a plan to take exactly one of each type of pill without wasting any. What is your plan?

12 Mailing a ring (2)(-)

Dan and Maria have fallen in love (via the internet) and Dan wishes to mail her a ring. Unfortunately, they live in the country of kleptopia where anything sent through the mail will be stolen unless it is enclosed in a padlocked box. Dan and Maria each have plenty of padlocks, but none to which the other has a key. How can Dan get the ring safely into Maria's hands?

13 4 Coins on a Rotating Table (4)

Charlie has in front of them a table that rotates as well as 4 coins on it that are each hidden underneath a cup. The game is as follows, Charlie will choose two cups to look under and then choose to flip zero, one, or two of the coins that they see. The cups are then replaced and the table is rotated some random distance. None of the cups or their positions are distinguishable to Charlie. Charlie will then repeat the process of choosing two cups. The game immediately ends if all 4 coins are facing heads up or all 4 coins are facing tails up. Can Charlie come up with a strategy to always end the game within 4 actions?

14 Scales (2)

You have 7 balls of equal weight and 1 ball that is slightly heavier. You have a set of scales with which you can compare weights of 2 sets of balls to one another. The scale will break after 2 uses. Find which of the balls is the odd one out.

15 More Scales (2)

You have 12 balls, one of which is a slightly different weight, possibly heavier or possibly lighter. You have a set of scales with which you can compare weights of 2 sets of balls to one another. The scale will break after 3 uses. Find which of the balls is the odd one out and whether it's heavier or lighter.

16 Truthful and Lying Statues

16.1 As represented in David Bowie's Labyrinth (2)

You approach two indistinguishable doors and two indistinguishable statues. One of the doors will bring you to riches while the other hides certain doom. Thankfully, the two statues are able to communicate and know exactly which of the doors is which. The issue is one of the statues always lies, while the other always tells the truth. You are allowed to ask one of the statues one yes or no question to figure out which of the doors is safe.

16.2 A whimsical statue? (3)

There is a third statue which, according to it's whimsical nature, arbitrarily decides to lie or tell the truth in response to any given question. You are now allowed to ask two questions to the statues to figure out which door is safe. You may ask the same statue multiple questions. The statues categorically refuse to answer any question that might lead to a paradox.

16.3 Ja vs. Da (5)

Once more, there is a third statue which arbitrarily decides to lie or tell the truth. But now, the statues speak their own language that you don't understand and they respond to your questions with "ja" or "da"(yes and no in their language). Once more, if a question is unanswerable by one of the statues, none of the statues will answer that question. The statues do not know how the whimsical statue will answer. You may ask the same statue multiple questions. The statues categorically refuse to answer any question that might lead to a paradox. With 3 questions, can you discover which statue is which?

16.4 Imploding heads (5)

The setting is the same as the previous problem. However, if a statue is asked a question that they can't answer because of paradoxes, they instead implode. The statues are also now willing to answer questions that lead to paradoxes. You may ask the same statue multiple questions. With 2 questions, can you identify which statue is which?

17 Cross a bridge (2)

Four people are crossing a rickety bridge in a dark cavern, so they all need a torch to cross. They only have a single torch that lasts 15 minutes. Alice can cross in one minute, Ben in two minutes, Cindy in five minutes and Don in eight minutes. The bridge cannot hold more than two people at a time. How do they get everyone across?