

## 113 mid-term: The Pentagon and Hexagon Groups

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Consider  $C_n$ , the cyclic group of order  $n$  (isomorphic to the integers mod  $n$  under addition);  $S_n$ , the group of permutations of  $n$  points; and  $D_n$ , the group of symmetries of a regular  $n$ -gon.

In a question "XXX is isomorphic to \_\_\_\_\_", the answer is one of  $C_n, D_n, S_n$ . Complete:

1. The cardinality of  $C_n$  is \_\_\_\_\_.
2. The cardinality of  $S_n$  is \_\_\_\_\_.
3. The cardinality of  $D_n$  is \_\_\_\_\_.
4. The largest cyclic subgroup of  $D_n$  has cardinality \_\_\_\_\_.
5. The center of  $C_n$  has cardinality \_\_\_\_\_.
6. The derived subgroup of  $C_n$  has cardinality \_\_\_\_\_.
7. The center of  $D_5$  is isomorphic to \_\_\_\_\_.
8. The derived subgroup of  $D_5$  is isomorphic to \_\_\_\_\_.
9. The center of  $D_6$  is isomorphic to \_\_\_\_\_.  
The quotient  $D_6/Z(D_6)$  of  $D_6$  by its center is isomorphic to \_\_\_\_\_.
10. The derived subgroup of  $D_6$  is isomorphic to \_\_\_\_\_.
11. The biggest  $n$  such that  $C_n$  is a subgroup of  $D_n$  is \_\_\_\_\_.
12. The biggest  $n$  such that  $C_n$  is a quotient of  $D_5$  is \_\_\_\_\_.
13. The biggest  $n$  such that  $C_n$  is a quotient of  $D_6$  is \_\_\_\_\_.
14. Can  $C_6$  be a subgroup of  $S_5$ ? \_\_\_\_\_  
Explain in at most 2 lines.
  
15. Can  $C_4$  be a quotient of  $S_4$ ? \_\_\_\_\_  
Explain in at most 2 lines.
  
16. Complete, and explain in at most 2 lines:  
"Let  $G$  act on a set  $X$ . The stabilizer of a point  $x$  is equal to  $G$  if and only if  $x$  is a \_\_\_\_\_."