MIDTERM EXAM, MATH 140, 10/17/2005 D. Geba

① a) Show that the parametrized surface
$$X(u,v) = (v\cos u,\ v\sin u,\ au),\quad a \neq 0$$
is regular.

b) Compute its normal vector $N(u,v)$

c) Using eventually b), prove that the angle formed by the tangent plane with the z axis along the coordinate line $u = u_0$ is proportional to the distance from the corresponding point $X(u_0, v)$ to the z axis.

② a) Show that $X : (0,+\infty)\times(0,2\pi) \longrightarrow \mathbb{R}^3$
$$X(u,v) = (u\sin\alpha\cos v,\ u\sin\alpha\sin v,\ u\cos\alpha),$$
where α is const., is a parametrization of the cone with 2α as the angle of the vertex.

b) In this coordinate neighborhood, prove that the curve
$$\alpha(t) = X\left(c\cdot e^{t\sin\alpha\cot\beta},\ t\right)$$
where c, β are const., intersects the generators of the cone ($v = $ const.) under the angle β.

③ Let S be a regular surface covered by two coordinate neighborhoods V_1 and V_2, for which $V_1 \cap V_2$ has two connected components W_1 and W_2.

The Jacobian of the change of coordinates is positive in W_1 and negative in W_2. Prove that S is nonorientable.