Mathematics 130, First Midterm

September 28, 2005. Professor H. Wu

Your Name: ______________________________

1. (15%) Let A, B, C be collinear points so that $A * B * C$. Then there is a *positive* number λ so that $A - B = \lambda(A - C)$. (*You may assume that the line containing A, B, C is neither vertical nor horizontal.*)

2. (20%) If $F : \mathbf{R}^2 \to \mathbf{R}^2$ is an isometry, then F maps a segment $\overline{PQ}$ onto the segment $\overline{P'Q'}$, where $P' = F(P)$ and $Q' = F(Q)$.

3. (20%) Given two lines $L = \{ax + by = c\}$ and $L' = \{a'x + b'y = c'\}$. Then $L \perp L' \iff aa' + bb' = 0$. (*You may assume that* a, b, a', b' *are all nonzero.*)

4. (20%) Prove this part of the C-S inequality: Let P, Q be two points in $\mathbf{R}^2$, $Q \neq 0$, so that $|\langle P, Q\rangle| = |P|\,|Q|$. Then P is a multiple of Q. (*You may assume that* P *and* Q *are distinct and the coordinates of* P *and* Q *are all nonzero.*)

5. (25%) Let $D = (d_1, d_2)$ be a point in the interior of $\angle AOC$, where O is the origin and $A = (a_1, a_2)$ and $C = (c_1, c_2)$. Then A and C are on opposite half-planes of the line OD.