Math 113: Midterm 1
October 4, 2005
Prof. Beth Samuels

Name :

Student ID Number:

Instructions: This is a closed-book test. Each problem is worth 20 points. Read the questions carefully, and show all your work. All work should be done on the exam paper. Additional white paper is available if needed. Good luck.

Problem	Score
1	
2	
3	
4	
5	
6	
Total	

(1) Prove that the identity element of a group G is unique.

(2) (a) Define a **subgroup** H of a group G.
(b) Let G be a finite or infinite group. Let $a \in G$. What is the smallest subgroup of G that contains a. Explain.

(3) Show that any group with four elements is abelian.

(4) (a) Define an **isomorphism** between two groups G and G'. (Define all terms you use.)
(b) Show that the additive groups $\mathbb{Z}$ and $5\mathbb{Z}$ are isomorphic.

(5) (a) Define a **quotient group** of a group G.
(b) Let G be an abelian group and let H be any subgroup of G. Is G/H a quotient group? Prove your answer.

(6) Show that any group homomorphism $\phi : G \to G'$, where $|G|$ is prime, must either be the trivial homomorphism ($\phi(g) = 1$ for all $g \in G$) or an injective ($1-1$) map. (You may use statements we proved in class.)