MATH H110

PROFESSOR KENNETH A. RIBET

First Midterm Exam

September 29, 2003

12:10–1:00 PM

Name:

SID:

Please put away all books, calculators, electronic games, cell phones, pagers, .mp3 players, PDAs, and other electronic devices. You may refer to a single 2-sided sheet of notes. Please write your name on each sheet of paper that you turn in; don't trust staples to keep your papers together. Explain your answers in full English sentences as is customary and appropriate. Your paper is your ambassador when it is graded.

Problem:	Your score:	Total points
1		10 points
2		10 points
3		10 points
Total:		30 points

1. Suppose that F is the field of rational numbers. Let $V = \mathsf{P}_{100}(F)$ be the F-vector space consisting of polynomials over F of degree ≤ 100. Let $T = \dfrac{d}{dx} : V \to V$ be the differentiation operator $\sum_{i=0}^{n} a_i x^i \mapsto \sum_{i=1}^{n} i a_i x^{i-1}$. Find the nullity and the rank of T.

Suppose now instead that F is the field Z_5 consisting of the integers 0, 1, 2, 3 and 4 mod 5. What are the nullity and the rank in this case?

2. Let V and W be vector spaces over F, with V finite-dimensional. Let X be a subspace of V. Establish the surjectivity ("onto-ness") of the natural map $\mathcal{L}(V, W) \to \mathcal{L}(X, W)$ that takes a linear transformation $T\colon V \to W$ to its restriction to X.

3. Let V be a finite-dimensional vector space over F. Let V^* be the vector space dual to V. Let $T\colon V^* \to F$ be a linear map. Show that there is a vector $x \in V$ such that $T(f) = f(x)$ for all $f \in V^*$