MATH 105 - MIDTERM 3/10/2006   D. Geba

1. Prove that for A, B and C measurable sets, the following identity holds:

$$\mu(A) + \mu(B) + \mu(C) + \mu(A \cap B \cap C) = \mu(A \cup B \cup C) + \mu(A \cap B) + \mu(B \cap C) + \mu(C \cap A)$$

2. Let $E \subset \mathbb{R}$ be a nonmeasurable set and define $f : \mathbb{R} \longrightarrow \mathbb{R}$ by

$$f(x) = \begin{cases} x, & x \in E \\ -x, & x \in E^c \end{cases}$$

Investigate whether f is measurable or not.

3. Define $f : [0,1] \longrightarrow \mathbb{R}$ by

$$f(x) = \begin{cases} 0, & x \in [0,1] \cap (\mathbb{R} \setminus \mathbb{Q}) \\ \frac{1}{q}, & x = \frac{p}{q} \in [0,1], \, q > 0, \, (p,q) = 1 \end{cases}$$

Prove that f is integrable and compute $\int_{[0,1]} f \, d\mu$.